Randomness, Information Theory, and the Unknowable

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Announcer: Very few people throughout history have made such landmark contributions that they have spawned entire fields of study. So when you have the opportunity to talk with one, you take it and Gregory Chaitin co-founder of the field of algorithmic information theory and all around swell guy is no exception. Join us as our host, Robert J. Marks dives deep with Gregory Chaitin into the depths of randomness, information theory and the unknowable today on Mind Matters News.

Robert J. Marks: There are few people who can be credited without any controversy with the founding of a game changing field of mathematics. We are really fortunate today to talk to Gregory Chaitin who has that distinction. Professor Chaitin is a co-founder of the field of Algorithmic Information Theory that explores the properties of computer programs. Professor Chaitin is the recipient of a handful of honorary doctorates for his landmark work and is a recipient of the prestigious Leibniz Medal. Professor Chaitin, welcome.

Gregory Chaitin: Great to be with you today.

Robert J. Marks: Thank you. Algorithmic information theory, when people first hear the term, at least at the lay level, the reaction is the field might be pretty dry and boring, but I tell them that algorithmic information theory, if you understand it is more mind blowing than any science fiction I've ever read or watched. So I hope in our chats we can convey some of the mind blowing results, bringing out of algorithmic information theory. So before diving in, I hope you don't mind if I get a little personal, your life has changed recently, you have two new children in your life.

Gregory Chaitin: Oh yeah.

Robert J. Marks: Juan, and how did I do in the pronunciation? That's a rough name to pronounce. Yeah. It's...

Gregory Chaitin: Yeah. Actually in Spanish, it's Juan. John in English, Juan in Spanish and the Portuguese pronunciation is a little different. It's more nasal it's Schwann.

Robert J. Marks: Schwann. Okay.

Gregory Chaitin: Yeah. And Juan Bernardo who's three. And our little girl is going to be 11 months. Her name is Maria Clara.

Robert J. Marks: That's wonderful. Now this has been a life change for you, hasn't it?

Gregory Chaitin: Oh, sure. For anybody who has children, it's a life change, isn't it?

Robert J. Marks: Yes, it is. I think we were sharing an email that I babysit for my grandson. And when I'm finished, I feel successful if I've kept him alive. He gets into all sorts of problems. He likes to pull things from the shelf on him. He climbs the stairs, and I'm afraid he'll fall down and need intensive care. And it's just one thing after another. So I expect you've experienced similar stuff. Right?

Gregory Chaitin: You bet. Actually, now that I'm a father, it seems like a miracle that children survive.

Robert J. Marks: Yes. It's amazing. Leonhard Euler, if I remember when he lost the sight of one of his eyes came out with a positive statement, he said, this is good because I have less to distract me from my mathematics. And I'm sure, but in a much more positive sense, this has kind of distracted you from your intellectual pursuits because... It's not a distraction, it's a wonderful addition to life, but have you found that to be the truth?

Gregory Chaitin: Well, that's a good question. I've worked a lot on mathematics and I've sort of done what I had intended to do. So I welcome this new project.

Robert J. Marks: New project. I read Virginia's comment in your new book, which we're going to post a link to it on the podcast notes, but she said that your children have completed your marriage's incompleteness problem. I thought that was hilarious. That really cracked me up.

Gregory Chaitin: Yeah. That was very clever of her, wasn't it?

Robert J. Marks: That was very clever. I think that our audience, much of our audience might not get the humor in that. But I think maybe as we go on, we can probably get this.

Gregory Chaitin: It's a joke for logicians.

Robert J. Marks: Yes, it is. It was funny. It cracked me up. I had a...

Gregory Chaitin: Yeah, it's a wonderful experience having children. It's true that both of us are older than normal, my wife and I. In my case, substantially older than normal, my wife, not that much. But we wanted very much to have children and it's a gift from God. We managed somehow.

Robert J. Marks: That's wonderful.

Gregory Chaitin: As a doctor said in the middle ages, I believe, he said something like I attended the patient and God cured him. So...

Robert J. Marks: Exactly.

Gregory Chaitin: ...in spite of all our efforts, it might not have worked, but we're very happy to have our two little children.

Robert J. Marks: It is. Yeah, it is wonderful. I think it says in Psalm 139, that we are fearfully and wonderfully made. And boy, that certainly is the case, isn't it? Speaking of youth, you did a lot of amazing stuff in your youth. I think you were in the Bronx, New York, is that right?

Gregory Chaitin: Well, we were living in Manhattan, a very nice location. Madison Avenue, a block from Central Park.

Robert J. Marks: Oh my goodness.

Gregory Chaitin: In the mid '60s, 68th Street, between 68th and 69th on Madison Avenue is a very nice neighborhood still.

Robert J. Marks: But I think I read, you went to the Bronx High School though, right?

Gregory Chaitin: I did go to the Bronx High School of Science, which was a treat. And I was also at the same time, I passed an exam in my first year at the Bronx High School of Science. I learned that there was something called the Science Honors Program at Columbia University, which was a weekend activity for very bright students interested in the sciences. And I got in, and one of the privileges I got as a result was they gave me the run of the stacks at Columbia University, which is unbelievable. Yeah. So for example, I was able to hold volumes of the collected works of Euler in my hands.

Robert J. Marks: Wow.

Gregory Chaitin: Yeah. And normal students, I don't think are allowed to go in the stacks, at that time they weren't. So this was an incredible treat. And another thing that happened was, I was 15, I came up with the idea of program-size complexity, and defining randomness in an essay question in the exam to get into the Science Honors Program at Columbia University.

Robert J. Marks: Interesting. I know in your work that you looked at things like relativity, this was in your teens for Pete's sake and quantum mechanics. So what led you down the path of computer science and to do the founding of algorithmic information theory?

Gregory Chaitin: Well, I was always very interested in computers. They were just starting at the Columbia Science Honors Program. They had a course where the kids could get access to computers. So I started programming in an assembly language. I think also in FORTRAN, there was a course given by a nice professor and they let the kids run programs on big mainframes, which is what we had then. So I was programming in high school, which at that time was pretty unusual, right?

Robert J. Marks: Yes.

Gregory Chaitin: Now it's not I'm sure. I was reading *Scientific American*. At first, my mother would read it to me. I was reading it very young. For example, I remember vividly a 19, I'm not sure what year it was. Could it be 1958 or 1956 article "Gödel's Proof" with a fantastic photograph of Gödel where he looks angrily at the camera with a blackboard behind him.

Robert J. Marks: Oh, yes. And he has that silver streak of hair, I believe.

Gregory Chaitin: It's possible. And then in '58, I think it was a book called *Gödel's Proof* came out. This was Nagel and Newman, they had written the article in *Scientific American* and I saw that book when it got published, I was 11. I had permission to take out books from the adult section in the New York City Public Library. I had piles of books at home and I was reading, reading, reading physics, mathematics. It was a nice time to be a kid growing up in Manhattan.

Robert J. Marks: I guess, especially if you're interested in that sort of stuff. You said you had your hands on some of the original works of Euler.

Gregory Chaitin: Yeah.

Robert J. Marks: Do you consider him, I consider him maybe if not the greatest, but certainly the most prolific mathematician in history.

Gregory Chaitin: Yeah. They may still be publishing his collected works. When I was a kid, there were many, many volumes and there were going to be more. He's my favorite mathematician. As far as I'm concerned, he's the... It's sort of silly to rank people. People talk of Gauss as the, I don't know what they say, the prince of mathematics or something, as far as I'm concerned, it's Euler. If Gauss is the prince, then Euler is the king.

Robert J. Marks: Euler is the king.

Gregory Chaitin: Yeah. His papers are beautiful to read. He gives his whole train of thought and it's obvious. It looks obvious, only while you're reading Euler. Gauss writes very concise papers, they are very hard to decipher. But while you're reading Euler, you think, oh, I could have done this, but of course only Euler could have done this.

Robert J. Marks: Yeah. I understand when he lost the sight in both of his eyes, he would sit around in St. Petersburg with students and dictate his ideas to them because he wasn't able to see.

Gregory Chaitin: Yeah. But his productivity didn't go down.

Robert J. Marks: His productivity didn't go down. Just an astonishing mind.

Gregory Chaitin: Yeah. It's astonishing because you ask, where did all this creativity come from? New mathematics, beautiful new mathematics was just pouring out of his head onto the paper and the publishers couldn't keep up. So his papers... So he had a pile of papers that he had and every now and then somebody would take some of them off the top, I think. And he would keep adding more so they weren't published in the order that they were done. And when I was reading the collected works of Euler, the Russians had this pile of manuscripts that hadn't all been included in his collected works yet. And they were being cagey. They were going slow.

Robert J. Marks: So you suspect that the works of Euler are still being translated, you believe?

Gregory Chaitin: I don't know. This was what's 60 years ago. So maybe they finally... No, they were being published in the original language. Latin I didn't know, but I knew French so I could read his papers in French pretty much like I could read English, but a lot of it was Latin and then I had to struggle with a dictionary. But one knows the topic. If it was a paper on number theory, I knew some number theory. So it was an absolute treat. I also had the collected works of Niels Henrik Abel in my hands, a child prodigy who did some beautiful work. But Euler works on every possible topic. And so where does all this new mathematics, where does this creativity come from? It seems to have a supernatural source. There seems to be as if...

Robert J. Marks: Well, that's great. In fact, that's a topic I want to talk to you about later. Whether or not maybe the creativity might be, for example, non algorithmic, non-computable?

Gregory Chaitin: Yeah. Maybe, God was talking to him, Cantor thought that, because it's really hard to explain where all those new ideas came from. Ramanujan is another example like that.

Robert J. Marks: Oh, he was that Indian that knew incredibly intuitive things about number theory, which were just mind blowing.

Gregory Chaitin: He didn't have a formal education in math. And he said that there was a Hindu goddess. I can't recall her name. He said that she would tell him mathematics while he slept.

Robert J. Marks: Really? Okay.

Gregory Chaitin: That's the most reasonable explanation I can think of for how he did that or how Euler did that. Unless we work out a complete artificial intelligence and it can do what Cantor or Ramanujan did. For now, I think the best explanation is the one that Ramanujan gave.

Robert J. Marks: Which was a supernatural sort of intervention.

Gregory Chaitin: Right.

Robert J. Marks: That's very interesting. Now, you mentioned that you read Gödel as a young man, the article about his incompleteness theorem in *Scientific American*. I also know that you had a near brush with Gödel and I've heard the story from you, but I've never seen it published. I wonder if you could share that near meeting with Gödel? That was fascinating, I thought.

Gregory Chaitin: I think it's somewhere maybe in a paper based on a lecture. Well, the story was, I had been in Argentina for a number of years and IBM sent me to the US. What happened was I was invited to be a summer visitor at the IBM Watson Research Center. And I was living in the YMCA in White Plains. And I'm not sure, this was a long time ago. So anyway, what happened was I had the proofs of one of my first papers on incompleteness. It was from the *IEEE Transactions on Information Theory*. This was in the early 1970s and I sent him the proofs. Well, I called him up, I looked up Gödel's phone number in the telephone book. I called him up. I think he picked up the phone and I said, "Professor Gödel, I have a different approach to proving incompleteness."

Robert J. Marks: So you cold called him then?

Gregory Chaitin: Yeah. Out of the blue. Instead of basing it on the paradox of the liar or Epimenides paradox, my approach is based on the Berry paradox and Gödel answered, "Well, it doesn't matter which paradox you use." He had said that in his 1931 paper, I was familiar with this paper. So I said, "Of course, but this suggests to me," I don't remember what I said, something like "a new approach that I would very much like to talk to you about and get your reaction." So he said, "Okay, send me a paper of yours on this topic. I'll take a look at it. And if I like it, maybe I'll give you an appointment." So I sent him the proofs. I had the proofs of that IEEE paper. It was my second *IEEE Information Theory Transactions* paper actually. Did it come out in '74?

Gregory Chaitin: Okay. So I sent him the proofs, it hadn't been published yet and I called him up and I think I remember he said, "Very interesting. Your notion of complexity is an absolute notion." Now this was a distinction he made between the idea of... What you can compute is absolute. It doesn't depend on the axioms, whereas what you can prove does. So he had taken a look at it and immediately perceived a crucial aspect of the definition of complexity that I was proposing. And he gave me an appointment. This is when I was visiting the Watson Center. So I did some research to figure out how I could manage without a car. I would take the train into New York City from Yorktown Heights. I would take the train out to, I don't know, Princeton Junction or something. I would get there! Nothing could stop me, right?

Gregory Chaitin: I was all set for the great day and it snowed. And this was the week before Easter. So that's unusual, a Spring snowstorm, but nothing was going to stop me. It wasn't a big snowstorm. Nothing was going to stop me from visiting my hero. So there I am in my office at the IBM Watson Center about to leave. I figured out how much time I needed. About to leave and unfortunately, very unfortunately, the phone rang. It was Gödel's secretary saying Gödel is very careful with his health and because it snowed, he's not coming into his office today and therefore your appointment is canceled. So that was a surreal experience. And there was no way to reschedule because I was going to leave just in a few days, heading back to Argentina, to Buenos Aires. But this surreal story actually fits better Gödel and his legend because for example, when Gödel died they found lots of answers typed up to letters he received, but that were never sent. They were never mailed. So there was a surreal quality to Gödel and to communicating with Gödel.

Robert J. Marks: He was a quirky guy and a germanophobe, if I recall correctly.

Gregory Chaitin: Yeah. He was from what, at the time, was the Austro-Hungarian Empire. And he didn't accept being made member of the Austrian Academy of Sciences. He never went back to Europe, never visited Europe. He turned it down, they offered him to be a member of the Austrian Academy of Sciences. So he's an interesting guy. One of the books I like about Gödel is in French, it's called *Les Démons de Gödel. Logique et folie*, so that means *Gödel's Demons: Logic and Madness*. And this was by a gentleman in France who actually went through the Gödel archive at Princeton. Half the book was also devoted to Emil Post, a forgotten genius.

Robert J. Marks: Oh yes. Emil Post. One of the things that I'm familiar with that Gödel did that wasn't published until after his death was his ontological proof of the existence of God based on Anselm's arguments. So I guess there were a lot of things of that sort.

Gregory Chaitin: Yeah. He was definitely a theist. He was living in the Middle Ages, sort of. There's a wonderful story that Rebecca Goldstein published. The Princeton Institute for Advanced Study would occasionally have fancy dinners, basically for people who might contribute additional funding to the Institute or provide political support. And at these dinners, they would ask their stars to be present to impress the potential donors and other members of the Institute. So Gödel was at such a dinner. You had to go, right? It wasn't optional. He probably wouldn't have wanted to be there. He was at a dinner sitting next to a young astrophysicist. And the astrophysicist was very proud of some discovery he had made. An observational discovery and he spent a lot of time explaining it to Gödel. And finally he stopped expecting Gödel to express admiration for the story he told. Instead of which Gödel replied, "I don't believe in empirical science. I only believe in *a priori* truths." That's an answer from the Middle Ages. *A priori* truths are necessary truths that don't depend on... So maybe he didn't believe in evolution either. I'm not sure.

Robert J. Marks: Well, he doesn't, there's a famous quote that he said evolution is... I forget the analogy, but it was basically that a printing factory explode and result in a book or something like that. So he was not a big believer in evolution either.

Gregory Chaitin: Yeah. So he was a very interesting person, an interesting mind.

Robert J. Marks: We are chatting with Gregory Chaitin and the co-founder of the field of algorithmic information theory that explores properties of computer programs. Professor Chaitin, welcome again.

Gregory Chaitin: Thank you very much, Bob. Yeah. And particularly it looks at the size of computer programs in bits and more technically, you ask what is the size in bits of the smallest computer program you need to calculate a given digital object and that's called the program size complexity or the algorithmic information content.

Robert J. Marks: And I've heard you call those *elegant programs*.

Gregory Chaitin: Well, the elegant programs are the... Yeah, that's the smallest program that has the output that it does. And its size in bits will be the measure of complexity or algorithmic information content.

Robert J. Marks: Yes, yes. Now this goes back to your co-founding the area of algorithmic information theory. And as a teenager, you published a landmark paper. The title of it was "On the Length of Programs for Computing Finite Binary Sequences." You published it in the *Journal of the Association for Computing Machinery*, which I guess is the oldest journal associated with theoretical computer science. It was founded in 1947, right after World War II. And this was a landmark paper in the founding of algorithmic information theory and you covered a number of topics. And one of them, which is just fascinating, is a brand new concept of the idea of randomness. You offered a whole new definition of randomness. Do you have a definition for randomness in general?

Gregory Chaitin: Well look, the normal definition of randomness is if the process that produces something is unpredictable, like tossing a coin. If you have a fair coin and you keep tossing it, that's going to give you a random sequence of heads and tails, right?

Robert J. Marks: Yes.

Gregory Chaitin: You look at how it was generated, the sequence. But what I wanted was a definition that doesn't... I wanted a mathematical definition because you see if you toss a coin, you could get all heads and that isn't random, but it is possible. So I didn't like that definition. So I wanted a definition of lack of structure. You see with the normal coin tosses, actually every possible finite sequence of heads and tails in a sense is equally random because they were all generated by tossing a fair coin, independent tosses of a fair coin. But some of them... all heads has a lot of structure.

Gregory Chaitin: All tails has a lot of structure. Alternating heads and tails have a lot of structure. So I was looking at something that ignored how the sequence is generated and just looked at it and said, is there structure here or isn't there? Now the reason for doing this is because you can think of a physical theory to explain a phenomenon as a program, a software that can calculate the predictions. So if the program is short, then you have a very comprehensible theory and a lot of structure. But if the program is the same size in bits as the number of bits of experimental data, then that's not much of an explanation. It's not much of a theory because there always is a program the same size in bits as the bits of data. Why? Because you just put the data into the program directly and print it out. That can always be done.

Gregory Chaitin: But the smaller the program is comparing its size in bits to the number of bits of data that you're trying to explain, and I'm talking about an explanation that gives no noise. It's not a statistical theory. It has to give every bit correctly in the data. If that's a small program, then you have a good theory. And if you have two theories and one of them is a smaller program than the other, the smaller program is a better theory, if two of them calculate the exact sequence of your experimental data. So it's sort of a model of the scientific method. Now I'm not using equations. Normally people talk about... There's a lot of talk about complexity in discussions of the philosophy of science, but they're talking about the complexity of equations, for example. And that's very hard to define and make a mathematical theory about it, because mathematical notation changes.

Gregory Chaitin: But if you have to explain to a computer how to calculate the observations, there are universal Turing machines, there are optimum computers that give the smallest programs and that's a good basis for a mathematical theory, a more precise definition of complexity. See, so when I was a kid, I was reading a book by Karl Popper called *The Logic of Scientific Discovery*, I think it was called. He has a whole chapter on simplicity and he points out some remarks of Hermann Weyl on this, another book that I read. Hermann Weyl, he was a student of Hilbert. He was a wonderful mathematician and mathematical physicist. And he wrote two books on philosophy where he says that the notion of causality in a theory, really saying that there's something that is governed by a scientific law, is meaningless unless you have a notion of complexity because otherwise there's always a law.

Gregory Chaitin: You can always find, he points out... This goes back to Leibniz in 1686, you can always find an equation passing through points of data on a graph. There's a thing called Lagrangian interpolation, which will produce an algebraic equation that passes through any finite set of points. So, Leibniz makes a similar remark that Weyl was aware of. So you have to have a notion of complexity as well as a notion of what a law is, because otherwise it's meaningless to say that there's a theory for something. I think this is a very deep remark. And the question was... I think Weyl also says, it's tough to define this precisely because mathematical notation changes. What are you going to use? Are you going to use Bessel functions in your equation, for example? They change as a function of time.

Gregory Chaitin: So it seems a bit arbitrary. Now, taking a universal computer and looking at the size in bits of a program gives a more definite notion of complexity that you measure in bits. Also, there's a problem because if you look at what Weyl discussed and what Leibniz discussed, they're talking about points of data that a scientist has on graph paper, and these points are infinite precision information. They're real numbers, right? In theory, a point is infinite precision, so it's an infinite amount of information. So a key step in algorithmic information theory is that you replace the original problem, which was points of data on graph paper and an equation passing through those points, which doesn't work out too well, although it's closer to the real case in real physics. You replace it by discrete and finite amounts of information.

Gregory Chaitin: So, you think of the scientific data you're trying to explain as a finite sequence of zeros and ones, and then the program, which is your theory is also a finite sequence of zeros and ones, in binary. And inside computers it's always binary. So then it's very easy to compare how many bits in your theory versus how many bits in your data. So the simplest theory is the best and if there is no theory simpler than the data you're trying to explain, then the data is random. It has no structure because a sequence of bits, you can always calculate it from a program the same size in bits as the data. So, that doesn't enable you to distinguish between a sequence of bits with structure, from a sequence of bits that has no structure. It's when you say that the program has to be simpler than the data you're trying to explain. Your theory has to be simpler than the data. Then it's a theory.

Gregory Chaitin: So this idea goes back to the *Discourse on Metaphysics*, which is a relatively short text of Leibniz. The original is in French, it's called *Discours de métaphysique*. It was found nearly a century after Leibniz died among his papers. The person who found it gave it this name and Weyl following the Germanic tradition had studied a lot of philosophy. Leibniz is a hero in the German speaking world. He's less known outside the German speaking world. And so his two books on philosophy mentioned Leibniz and mentioned this idea of Leibniz. Then Popper refers to it in *The*

Logic of Scientific Discovery, which was originally in German also, by the way. He was a refugee from the Second World War. And algorithmic information theory takes up this question and changes the context from a theory being an equation passing through a set of points which have infinite precision to making everything discrete and have a finite number of bits, and then mathematically you're in business.

Gregory Chaitin: So this was the fundamental idea that inspired, at least it inspired me to try to work out a detailed theory. And I had these papers in high school, but I did many versions of the theory and the one that I regard as definitive, it's called "A Theory of Program Size Formally Identical to Information Theory," also published I think it was 1975 in the *ACM Journal*. So the original versions had some problems and I think the definitive version is from 1975. Now, my interest in this was philosophical. I wanted to understand what a theory is, how to measure its complexity. But mostly I was interested in incompleteness because it turns out that this notion of complexity asked for the size of the smallest program to calculate something. This is how you measure the algorithmic information content of that digital object.

Gregory Chaitin: It's a nice definition. You have a nice mathematical theory, but you can never calculate it. Well, except in a finite number of cases for very small programs. Everywhere you go you get incompleteness in this theory, things that you can define, but you can't calculate. So incompleteness sort of hits you in the face in this theory. And my main interest was in incompleteness, in trying to extend the work of Gödel and Turing that I had studied as a teenager on incompleteness. But there are other people who have more practical interests. And making use of this criterion that a good theory is a small one, you can apply that to predicting future observations by looking at the size of programs ...

Robert J. Marks: This is kind of the work of Solomonoff, right?

Gregory Chaitin: Yeah. He was interested in prediction. I was more interested in looking at a given string of bits and asking, does it have structure or not? And the incompleteness results regarding this question. For example, most strings of bits have no structure according to this definition, they cannot be compressed into a smaller program, but it turns out you can almost never prove it. You can show that it's very high probability, but can only be provable for extremely small sequences. So that was what fascinated me, but Solomonoff was interested in artificial intelligence and Marvin Minsky praised Solomonoff's theory. And about a year before Marvin died, he was in the World Science Festival in New York City. Marvin and I were on a panel and we were filmed with Rebecca Goldstein and Mario Livio and a Nobel Prize winner in biology was the moderator.

Gregory Chaitin: And it's an hour and a half on film and at the very end, Marvin surprised me by saying that in his view, the decisive step forward from Gödel is using this approach to making predictions. Now he says it can't be done, it would require an infinite amount of computing to get precisely the best prediction according to this criterion. But he suspects there are good approximations and people ought to work on that. And in fact, indeed people have worked on that. Hector Zenil has done a lot with using approximate versions of these ideas that are computable.

Robert J. Marks: Yes. I'm an engineer that loves mathematics, and I teach a graduate course on information theory, including both Shannon and algorithmic information theory. And I explain the randomness in this fashion and let me pass it by you just to make sure that I'm explaining it right. It's the maximal degree to which a sequence of bits can be compressed. And we talk about compressing files using Lempel-Ziv and zip files and PNG images where they compress the image in order to transmit it over a channel. They do that much like dehydrated food. You take the water out, you ship it, because the shipping is cheaper and then you hydrate it on the other side. But this Lempel-Ziv doesn't... I've tried it on a number of different images, like for example, an image and the scaled image, and it doesn't take, so clearly the Lempel-Ziv and the zip files that we generate are not the smallest. And so I make the case that there must be a smallest file that generates the random output and that is the concept of what you call elegant programs. Is that pretty accurate?

Gregory Chaitin: Yeah, of course. That's a very good way to explain it. I was interested in Shannon's theory of information and the noiseless coding theorem, and the coding with redundancy when there's noise. One of my first papers was in 1970 in the *IEEE Information Theory Transactions*. I didn't start there with the philosophy of science. That interested me, but I didn't think the readers of that article would be very interested. So I started with the Shannon transmitter/channel/receiver diagram and said, well, let's send the smallest program to calculate something. That's noiseless coding. That's going to be the most compressed version. And then at the other end, what you do is you run it... That's a kind of a universal scheme for compressing. That'll give you the best compression...

Gregory Chaitin:

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... scheme for compressing. That'll give you the best compression possible if you use a computer at the other end to get back the original message. You run the program and it gives you the original message. The only problem with this is you can't get the best program, the most concise, compressed form of the message according to this definition. It exists in the Platonic world of mathematics, but actually finding it is impossible, in fact, in general. So, that's the philosophically interesting part, but you can view it from an engineering point of view. And if you're interested in AI and making predictions and Marvin Minsky was and Ray Solomonoff was, then this is... It's a very interesting new approach and as Minsky said... Well, he likes to be provocative. He said, "Everybody should work on this to find practical approximations to this impossible task." The person I know who's done it best has been Hector Zenil and his collaborators. He's in Europe.

Gregory Chaitin: So, my interests were more in proving theorems, and in particular, proving incompleteness theorems and the light it sheds on the scientific method. It's metaphysics. What is a theory? What is the simplicity of a theory? What's a good theory? If you have two theories, which will you look at? That was my starting point, but I was reading Shannon. I was reading Turing. I was reading von Neumann on game theory. Actually, I had forgotten the definition of randomness that I had put in the essay question to get into the Columbia Science Honors Program, the Columbia university program for bright high school students. I remembered it when I read a footnote in von Neumann. His theory for certain games says, "The best thing to do is to toss a coin," and that's because... Well, it's a little long to explain.

Gregory Chaitin: He has a footnote saying, "Actually, does that mean there's only a theory of games in a world where there exists randomness?" And I said, "No, you can have a theory that..." You see, everyone will know the theory. So if the theory says toss a coin, the fact that everyone knows the theory doesn't mean that you're dead because your opponent will know what you'll do. But another alternative instead of tossing a coin is if it's an uncomputable sequence of moves that you should make. That way the theory could tell you to pick a unstructured sequence, and then it's not a contradiction because you and your opponent will know the theory, but he won't be able to use that knowledge against you. But unfortunately, you won't be able to, unless an oracle or God gives you an unstructured maximum program-size complexity sequence, you won't be able to do what the theory says you should do. But in theory, it shows that in a world without randomness, you could also have a theory that would be impractical, but it would tell you what would be the best thing to do.

Gregory Chaitin: So, it was that footnote, where von Neumann says, "There's the strange aspect of this theory that it seems to depend on quantum mechanics and the fact that the universe contains randomness," and then I think he says, "This could be discussed at greater length," but he leaves it there. And I said, "Oh, the random sequences." The unstructured sequences I had thought of in that answer to that essay question at Columbia University, that would also work. Now, how you get them, I don't know, but it would work. So I was playing with that idea. So I remembered the definition and then I started to work out the mathematics. So, that was the summer between my first year and my second year at City College. And then the Dean excused me from attending classes because I was working on this immense paper.

Robert J. Marks: That's something.

Gregory Chaitin: The Dean was a mathematician, by the way, was a professor of math at that time. I was at City College where Emil Post had been.

Robert J. Marks: Oh, okay.

Gregory Chaitin: They had his photograph on the wall of the office of the Chairman of the Math Department.

Robert J. Marks: Your test at Columbia reminds me of tests that I give. If you give a test where all the problems are simple, you get kind of a histogram with a little peak. If you make them all hard, you get another peak on the other end. So, an ideal test should have a gradient. And I tell the students that there's going to be some simple ones, some medium ones and some hard ones. Sometimes I ask questions which I don't know the answer for. So I tell them, "If you get the answer to some of the harder questions, we have a publication."

Gregory Chaitin: That's right.

Robert J. Marks: So I think that that's kind of what you did at the Columbia entrance test, right?

Gregory Chaitin: Yeah. Well, that reminds me of a joke by my late friend, Jacob Schwartz, a mathematician at Courant Institute. He floated the idea of putting Fermat's Last Theorem as a... including it in the problems in an important exam in mathematics, in the hope that some undergraduate would come up with a wonderful, short proof — Fermat claimed he had a short proof, a wonderful, short proof — not knowing that this was an immensely hard problem that many famous mathematicians had worked on unsuccessfully for a long time. But that's not how it was solved. It was done by a very fine, sophisticated mathematician working on it in secret for years, and it's a very long proof, Wiles' proof.

Robert J. Marks: Amazing. I guess Fermat was wrong when he said he could fit the proof in the margin.

Gregory Chaitin: That's an interesting historical question. When Fermat said he had a proof, he always had a proof, I think. The only case that was left hanging was that. So, he was a very superb mathematician, Fermat. So I personally think he had a proof, but we haven't figured it out. It was based on different ideas than Wiles' proof, because those concepts didn't exist at that time, but I could be wrong. By the way, there's a lovely musical comedy about all of this called *Fermat's Last Tango*, and it's available on the web. It's a conflict between the ghost of Fermat who doesn't want Andrew Wiles to find the proof, and Wiles' wife who would like Wiles to come back to earth, because he's working all the time on this in secret, and she doesn't get to see him very much nor do his children. It's great fun. It's a musical comedy. It's written by someone who knows mathematics so the jokes are all good math jokes.

Robert J. Marks: I got to ask, was it successful? It seems that the audience would be somewhat limited.

Gregory Chaitin: People were falling off their seats. It was wonderful. They're not terribly sophisticated math jokes, but they're all correct. The songs are correct and the history that they give. There's a song where Fermat is taunting Wiles: "Your proof has got a hole." It's very clever. It also has Heaven, where there are the ghosts of Euclid, Gauss, Pythagoras and Newton looking over all of this, and many different styles of music. It's great fun. They're dancing also. It's wonderful.

Robert J. Marks: I tell you, it takes a lot of talent to take something, a mathematical proof and make it into an entertaining play. That is wonderful.

Gregory Chaitin: It's a musical comedy which sounds even harder, right?

Robert J. Marks: Yes, exactly. So, we're going to find this. You say it's YouTube?

Gregory Chaitin: The Clay Institute, the one that has the Clay prizes of a million dollars for those very hard problems, the Clay Mathematics Institute, something like that, they paid the money to make a DVD and then somebody put it on YouTube. So you can now get it like that. I recommend it highly. Oh, and then there's a song taunting Wiles, where Fermat is taunting Wiles again to try to keep him from finding his proof, saying, "Mathematics is a young man's game and how old are you, Andrew?"

Robert J. Marks: Oh, did Fermat do this when he was older?

Gregory Chaitin: No, no, this is a fantasy. This is for Fermat's ghost. Oh, Fermat? I'm not sure. It's a marginal comment in a copy of Diophantus. I think the book was even lost. E. T. Bell thinks somebody stole it. So anyway, go and see this musical comedy. It's great fun and all the math is right.

Robert J. Marks: We will find the link to that and post it on the podcast notes.

Robert J. Marks: So, let me just start out with the question, is math discovered or is it invented? What's your take?

Gregory Chaitin: Well, I think that's a fascinating question, and deep philosophical questions have many answers, sometimes contradictory answers, even, that different people believe in. Some mathematics, I think, is definitely invented, not discovered. That tends to be sort of trivial mathematics papers that fill in much needed gaps because somebody has to publish. So you take some problem. You change the wording of the mathematical problem a little bit, and then you write a paper. You solve it and then you write a paper. But other mathematics does seem to be discovered. That's when you find some really deep, fundamental mathematical idea, and there it really looks inevitable. If you hadn't discovered it, somebody else would've discovered it because it really seemed to be a fundamental idea.

Gregory Chaitin: Now, so one idea is that mathematics is in the mind of God or in the Platonic world of ideas. It's all there, and all we do is discover it, but I think there's a distinction. Poincaré, a famous mathematician, Henri Poincaré, he said... It sounds better in French than in English. He said, "There are problems that we pose and problems that pose themselves." So those are the two different kinds, invented and discovered. So when you find some really fundamental new mathematical idea, you have this feeling that you've seen into the mind of God, and it's really fantastic. That would be Cantor, Georg Cantor. Or if you hadn't discovered it, someone else would have because it's so basic, it's so beautiful that it's got to be there in the mind of God or in the Platonic world of ideas. But everything is in the Platonic world of ideas, the Platonic world of mathematics. But if you are a mathematician at a university and you're struggling to publish I don't know how many papers per year, you can't work on such fundamental questions all the time, because then you won't publish enough papers, right?

Robert J. Marks: Yes.

Gregory Chaitin: So there is this pressure to invent stuff, minor variations on previous work, and that is a shame, I think, and I think it should be regarded as being invented, although one attitude is to say that it's all in the Platonic world of mathematics.

Robert J. Marks: Yeah. In engineering, we call those 3DB papers, three decibel papers, because three decibels is the minimal amount that you increase the volume of something and detect it. So, there are landmark papers and then there's lots of 3DB incremental papers...

Gregory Chaitin: I see.

Robert J. Marks: ...that you talked about.

Gregory Chaitin: Yeah. You guys have the same pressure to publish as everyone else, right?

Robert J. Marks: Exactly. Exactly.

Gregory Chaitin: Yeah.

Robert J. Marks: Yes, it's kind of unfortunate because you don't hear the word scholarship very much anymore in academia.

Gregory Chaitin: Yeah. And people don't write books. In the past, some wonderful mathematicians like G. H. Hardy would write wonderful books, *A Mathematician's Apology* or his book on number theory.

Robert J. Marks: And they're just beautiful papers. Claude Shannon's paper, it's just a wonderful paper in the founding of Shannon information theory. Lotfi Zadeh in 1965 wrote a wonderful paper on the founding of fuzzy or soft logic, and I don't see those sorts of papers anymore. Everybody's interested in number more than quality.

Gregory Chaitin: Yeah. Well, nobody can work on a difficult project, which may be years until you come up with something or maybe you'll never come up with something. So, you have to do that. If you want to try that kind of stuff, you have to do it in parallel with more normal stuff. I think that Andrew Wiles, when he was working in secret for years in his attic on Fermat's Last Theorem, he couldn't have stopped publishing altogether. So he probably had more routine mathematical questions he published on and he probably cursed them because it was taking time away from his great project, but the great project could fail, could have failed.

Robert J. Marks: He would need something to fall back on. There's an old academic joke that the Dean can't read, but the Dean can count. So, they don't look at the contents of the paper, but they just count the paper numbers. I think that that's unfortunate and it's a pressure to reduce the quality of papers.

Gregory Chaitin: Well, there also used to be professors who were wonderful teachers. The students adored them. They learned a lot from them, but they weren't research mathematicians, for example, and you can't do that anymore. You don't get credit for being a wonderful teacher, as far as I know, or for writing wonderful books. You have to have refereed papers, if I'm not mistaken. So, that's too bad, but what can you do? We have to play it according to the current rules, right?

Gregory Chaitin: I was able to not play it according to the current rules because I was being paid to do industrial development work for IBM at IBM Research. So I worked on hardware. I worked on hardware design. I worked on the software for new hardware. That was a lot of fun. I was doing my mathematical research as a hobby basically. So, that's how I created an ecological niche for myself, because a rather unconventional story as a mathematician, I don't have a degree. I only have honorary degrees.

Robert J. Marks: You hear about people who were discouraged from doing publication. I don't know if it was Popper or it was somebody like him that wrote that his institution didn't want him to publish because it took away time from teaching, and that certainly changed.

Gregory Chaitin: Oh, he was a professor at first in New Zealand, I think, and he had a terrible... He had to escape from Europe. I think he might have been Jewish and maybe he... Anyway, so he managed to get into New Zealand and he had a terribly heavy teaching load. And then fortunately, he managed to go to the London School of Economics, I think it was. And there, he didn't have that outrageously heavy teaching load.

Robert J. Marks: We even see this from... I guess he was an employee of the Guinness Brewing Company, Gosset, who came up with this incredible math, which is still taught today to undergraduates. He couldn't associate his name with the article so he published under the name Student T, and we still refer to it as the Student T or the T-statistic. He was also under pressure not to take time away from his work. Today, it's exactly the opposite, by the way. If you're a mediocre teacher and you publish a lot, you get all sorts of accolades, as long as your teaching is acceptable. That's unfortunate, but that's the way things have evolved.

Gregory Chaitin: Yeah. Well, what can you do?

Robert J. Marks: Yeah. You got to play the game, I guess, right?

Gregory Chaitin: Well, you don't have to. Look at Elon Musk. He's my great hero. He's a wonderful engineer and he's a wonderful entrepreneur and he doesn't follow the rules

Robert J. Marks: He doesn't, and innovators don't follow the rules. I think that's one of the elements and one of the characteristics of creativity.

Gregory Chaitin: And that's tough. I mean, Elon is clearly a genius, amazing engineer, incredibly talented and innovative, but he also has to figure out a way to do this in the real world and he has managed to do it. So, that's a remarkable achievement also. I admire him greatly.

Robert J. Marks: But then again, we were talking offline about the relationship between genius and creativity. And we talked about, for example, the quirkiness of Gödel, and Georg Cantor spending a lot of his life in a sanatorium because of mental anguish. Elon Musk has his quirks too. He has opined that we are all computer simulations.

Gregory Chaitin: He has? Well, that's a popular view now. The computer has replaced God in a lot of people's minds and I think we're all the poorer for it, but it's the fashion now. So, we're machines. We're machines. AIs are going to be better than us. Human beings will be obsolete. This is the fashionable view, and of course, I don't appreciate it very much.

Robert J. Marks: Well, that's wonderful. Mind Matters and our podcast is part of the Bradley Center, the Walter Bradley Center for Natural and Artificial Intelligence, and that's one of the things that we push back on, is the idea that there are things that machines can't do that humans will always be able to do. We actually use some algorithmic information theory to back the fury. So yeah, I'm glad to hear that. I'm glad you're not a proponent that machines are going to replace people. We still have some attributes that I think that will never be duplicated.

Gregory Chaitin: They might replace people if they're cheaper and better at uninteresting tasks, but I think human beings will always be better at creativity, at doing art. As Turing said, "A computer is likely to write poetry that only another computer would enjoy reading."

Robert J. Marks: That assumes that another computer can enjoy. I don't think computers have the capability of enjoying.

Gregory Chaitin: Yeah, I don't know. Elon Musk is worried that AI will get out of control. He's also has his personal project to not let computers replace humans by coupling computers and humans into a symbiosis, where both contribute what they're best at. And I can't remember the name of the company that's working on a higher bandwidth connection.

Robert J. Marks: It's called Neuralink, I believe.

Gregory Chaitin: Yeah, that's right. That's the one I'm thinking of.

Robert J. Marks: Where he implants a chip in the brain. And I think I'm going to wait awhile before I do that. I don't think I want a chip in my brain.

Gregory Chaitin: Yeah. To justify doing that, they're doing it for people who're quadriplegics, for example, who need help, that no one will argue that it's a good thing. But trying to make a symbiosis between humans and computers, Elon thinks you need to do that so that human beings aren't left behind. So if you give a high bandwidth link between computers and people, he thinks that will help people to not feel obsolete, but to sort of participate. Human beings use machinery and we don't think that just because I can't run as fast as a sports car or lift as much weight as a steam shovel, we don't think that that means that human beings are valueless. We invented those devices.

Gregory Chaitin: Similarly, some things computers certainly are better at than humans, like remembering precisely large quantities of information, unless you have a photographic memory, and Elon seems to have a photographic memory for technology. Von Neumann was said to have a more general photographic memory. So a symbiotic relationship between the two of us, each one might contribute what they're better at and people will not feel that they have become obsolete. It'll just be like using a steam shovel or using an airplane instead of trying to fly by flapping your arms.

Robert J. Marks: Well, the comedian, Emo Philips, says that computers might be able to beat him at chess, but he can always win a game of spirited kickboxing. So, I think that, yeah, there are things which computers can do, they can do well, but there are limitations on them.

Gregory Chaitin: On the other hand, we invented the computer, so we can take the credit for whatever they do well.

Robert J. Marks: Well, yes. And you'll notice this idea of Elon Musk's fear, and I don't want to detract from Elon Musk because he's clearly a genius, but this assumption that computer software is going to write more creative computer software, that it's going to write or create computer software, and you're going to have an AI which reaches just this hyper-intelligence, has the assumption that computers can write programs that are creatively more able to do things than the original computer programs.

Gregory Chaitin: Yeah, I would think a team of brilliant engineers might write an amazing piece of AI software, but the AI software doesn't rewrite itself.

Robert J. Marks: Exactly. This actually dovetails in... This is something I think I read that you wrote and you have to correct me if I'm wrong, but you were talking about computer programs and software that was able to prove meaningful theorems, that is insightful theorems of the type that a brilliant mathematician would write, and I believe you said that there's no evidence of that happening.

Gregory Chaitin: Well, I made that remark some years back. What they have now are proof checkers. You write the proof in a special language, precise mathematical notation. There's software that can check if the proof is correct or ask you to provide more steps if it doesn't understand how one thing followed from another. That technology is improving. So there are mathematicians who claim that at least that all of math should be written up this way and submitted to checking like this, but these computers are not doing wonderful new mathematics.

Robert J. Marks: Exactly. There's a difference between checking the proof and originating the proof.

Gregory Chaitin: Yeah, there's an enormous difference. Now, there is what I regard as a piece of AI. So it might be interesting to talk about it. My friend, Stephen Wolfram, the system he's created, I don't know what it's called the Wolfram language, Wolfram Alpha.

Robert J. Marks: Yes.

Gregory Chaitin: What Euler would've accomplished with that is unbelievable. Euler and Gauss used to calculate, they were wonderful at doing calculations and they would do lots of calculations, and then make conjectures based on the patterns they saw. Well, if Euler or Gauss had had Wolfram Alpha or Mathematica, they would've done a lot more, especially when you go to Wolfram Alpha, it begins to start feeling like an AI.

Gregory Chaitin: Now, it's an AI that has a big team of people behind it, who take information and curate it about the world, about physics, about chemistry, about economics, about geography. They curate it and they put it into this system, but it's pretty amazing. It would've looked like magic, I think, to people. Well, computers, almost any computer would've looked like magic just a few years ago, but I think this is a genuine AI, but it's not a human general intelligence and it's not creative, it's different, but I think it's a enormous achievement what Wolfram has done.

Robert J. Marks: Oh, Wolfram's Mathematica and his other works are just astonishing in what they can do, but as you mentioned, they're all algorithmic. The logical steps, much like the theorem checker, are something which humans have placed in there, which allow you to put in things like indefinite integrals and advanced calculus equations and it gives you the solution. It's really, really remarkable.

Gregory Chaitin: Yeah. Wolfram is a genius. I rate him with Elon Musk. He's a genius at different kinds of technology than Elon is. And so, Wolfram Alpha is an accomplishment of this man of genius, who is just like Elon. Elon has an enormous team of very talented engineers, but he's on top of the whole thing, making it work. Wolfram has wonderful mathematicians, wonderful software people working for him. So, this artificial intelligence of an inhuman kind that they've created, it's very powerful, but it's done by human beings, so I think we should be proud of that achievement.

Robert J. Marks: We should be proud.

Gregory Chaitin: But it's not creative. Yeah.

Robert J. Marks: Yes, I think the creativity is the big thing.

Gregory Chaitin: It didn't program itself. Wolfram worked very hard with all these people to make it capable of doing more and more and more. It wasn't his software that made this thing evolve to what it can do now. It was all of them working very hard on it and Wolfram making sure they had a system that could be extended because what often happens to software is that — I know because of my work doing software for IBM — is there comes a point where basically the software dies, because what happens is it's so complicated that no one can understand it anymore, which means if you get bugs, it's tough to debug it and it's also tough to make any enhancements. So the fact that the Mathematica language has gotten us all the way to Wolfram Alpha is something that Stephen worked very hard on to have a system that could grow and be extendable, that wouldn't end up trapping him in a corner, like most large corporate software does eventually. So far he's achieved this remarkably.

Robert J. Marks: Oh, it's astonishing.

Gregory Chaitin: Yeah. But this is a human being of genius with a very talented team of engineers, mathematicians. This is not software that reprogrammed itself.

Robert J. Marks: Well, I think that AI in general is going to be a tool, which we can use to better ourselves.

Gregory Chaitin: Absolutely. Like a steam shovel, right?

Robert J. Marks: Like a steam shovel. Exactly.

Gregory Chaitin: Doesn't mean that human beings are obsolete.

Robert J. Marks: Well, I read the chapter by Stephen Wolfram from in your tribute book, which we are also going to list in the podcast notes, and he went somewhere, I'm not sure where, but he went to a library and he took a bunch of pictures of the notes of Leibniz. And I tell you, boy, we've come a long way. These old mathematicians, they couldn't compute e to the third power. They just couldn't enter it. They had to go to their margins and work out all the details, and it's astonishing all of the work that they had to do that we don't have to do today.

Gregory Chaitin: Exactly. And Leibniz made mistakes in some of his arithmetical calculations there in the manuscripts. He wasn't good at that, but we don't... I don't know. You could say we don't have anybody at the intellectual level of Leibniz. No, it depends how you rank it because he was good at so many things. He came up with fundamental ideas, new ideas in so many fields. Maybe it's because he never married or never had children, but...

Robert J. Marks: Yes, exactly.

Gregory Chaitin: But he was off the scale, which shows what human beings can achieve. Euler and Ramanujan and Cantor show what human beings can achieve.

Robert J. Marks: Well, this is very... I was sitting down kind of tallying, I think, the intellectual giants that have introduced new mathematical ideas, brand new, and I was thinking of people like Claude Shannon, Lotfi Zadeh, yourself and I don't know if we see the introduction of new, great ideas today, at least I don't see them. Do you have any thoughts on that?

Gregory Chaitin: Yeah. Well, I think the bureaucracy is killing the golden goose. There's too much control. You have to get research funds. You have to publish lots of trivial papers. You spend too much time filling out grant proposals.

Robert J. Marks: Yes.

Gregory Chaitin: So they're managing to make it impossible for anybody to do any real research. You have to say in advance what you're going to accomplish. You have to have milestones, reports. And the European community has made it worse. I was talking to a scientist in Europe and she told me, "I have to spend all the time dealing, interfacing with the bureaucracy in Brussels. I put together a research team, but they're the only ones really doing the research because my time is all taken with this administrivia." So, if you give the bureaucrats a chance, they'll grow and grow and grow and eventually sink the ship, but this seems to be the way this society is working. The Chinese seem to be innovating in engineering in a remarkable way. They have a different system. I don't know what it's like there. I've seen videos of them putting up a building with amazing speed, for example.

Robert J. Marks: There's an old saying that only rich countries can afford poets. We used to have these great research centers such as Bell Labs, which dissolved after divestiture, I guess.

Gregory Chaitin: Yeah. They got so many Nobel Prizes. They got so many Nobel Prizes.

Robert J. Marks: Yeah, it was incredible, but they dissolved. It was a rich country so they could have these poets, where they got together some of the greatest engineers and scientists of all times. Maybe they would only make one big breakthrough in their lifetimes, but they employed them for their lifetimes for their scholarship. We also see this today at Google, where Google is making available to people all of this wonderful artificial intelligence software. And so, that's where I see the innovation coming from and not so much from academia.

Gregory Chaitin: Right. Well, the universities were always very conservative. Elon Musk makes, I think, this remark in an interview. Maybe it was just a few days before that remarkable flight of the Starship SN8. I think he makes the remark that if you don't have a lot of research projects that fail, you're not doing enough research. And the problem is if failure is unacceptable, then you're in trouble. So for example, the legacy aerospace companies that make rockets, they take years to design a rocket and it's got to work on its first flight, right? Whereas Elon does rockets, does rocket engineering, the way you do software, you develop the software as you are using it.

Gregory Chaitin: The software I worked on, we were constantly using our own software. We were doing a compiler and we kept compiling the compiler through itself. So we were constantly eating our own cooking. We had many prototypes. If there was something wrong, we would fix it and try again, and Elon is doing that. He's making his rockets very fast and breaking them and learning from each failure. So even if you have a lot of research projects, failure is now unacceptable. So that means that the research projects have to be very conservative. You can't try something really crazy, right?

Robert J. Marks: Yeah. If you try to do a project and you fail, you can't publish it. So, that's bad for the bean counters.

Gregory Chaitin: Yeah. Well, the bean counters should get out of our way.

Robert J. Marks: So, let me ask you this, what is the solution? Do we have any solutions? One of which, this is very controversial to me, I think that some of the government funding is not good. I know in Japan and Germany, they require... Well, this is one solution. They require interface of the professors with industries so that they can work on more interesting problems, but that doesn't clear people up to pursue pure creativity. So, what's the answer? How can we fix this?

Gregory Chaitin: It's tough. I have a pessimistic vision, which I hope is completely wrong, which is that the bureaucracies are like a cancer, the ones that control research and funding for research and counting how much you've been publishing. I think that I've noticed at universities, for example, the administrative personnel are gradually taking all the best buildings and expanding. So, I think that in a society, the bureaucracy and the rules and regulations increase to the point that it sinks the society. At that point, basically, I think I expect this with countries, the country will collapse because it'll fail in a competition with a younger, more vigorous, more daring country. So nations and corporations seem to have a life cycle like human beings do, vigorous youth when they think they can do anything. And then they get very conservative. They don't want to come up with a new product, which competes with their existing product line, because you can't predict how much it's going to earn in advance.

Gregory Chaitin: At IBM, this used to happen. The salespeople would... Suppose you have a completely new technology, a new kind of computer. They're going to make a very low estimate of how many are going to sell. So we have to charge a lot for each one because we had a lot of development costs and you have to divide it by the... We weren't allowed to dump products. So the result is that it's a lost cause. If you want to try something daring, a new product, it's going to be so expensive that no one is going to buy it.

Robert J. Marks: Yeah, it's frustrating.

Gregory Chaitin: It's more than frustrating. I think it's the end. When I said society reaches that point, their innovation is going to go down. I remember when I was a kid, *Scientific American* every month was very thick. Why was it thick? Because it had lots of ads from General Dynamics and from aerospace companies that were trying to hire wonderful engineers. The things were more dynamic. I mean, what did an airplane engineer say in one speech I heard? He said, "In the old days, a bunch of engineers could go to a motel for the weekend so they wouldn't be distracted by their family with a bunch of six packs of beer and design a new airplane." That doesn't happen anymore. So, Elon refers to some of these topics in...

PART 2 OF 4 ENDS [01:10:04]

Gregory Chaitin: Elon refers to some of these topics in an interview by *The Wall Street Journal*, with somebody at *The Wall Street Journal*. It was on the 8th of December. I found it on YouTube. Since I was working in industry, I could see all these forces at work at IBM, which in the early days was full of adventurers. There was no computer engineering. The guy I worked with, his field was English literature originally, but he wanted to make a new industry. He was fascinated by computers and he was one of the great contributors at IBM. So IBM was very vigorous in innovation at first, but then it got more and more bureaucratic and afraid of competing with their existing products. So you get to the point where a new computer can only come from a new company because the existing company will never want to take a chance on something new.

Gregory Chaitin: So anyway, all this worries me. I hope I'm completely wrong and this doesn't happen. Elon Musk certainly is an example that it's still possible to be tremendously innovative in the field of technology. But he also talks about bureaucracy and it's a question that worries him a lot. And the fact is that failure is not allowed, whereas you have to learn from your failures. If you don't fail, it means you're not innovating enough. So that worries me a lot. I hope I'm wrong, but I have this vision of the life cycle of corporations and nations. I saw what IBM was going through. And a lot of people are worried that China is becoming more capitalist than we are in a funny way. I don't know, but it's something that I worry about. And here in South America, I see US influence disappearing and more and more business with China. China is now the leading trade partner for a lot of countries in South America, rather than the United States.

Gregory Chaitin: I used to have an account here, a bank account in Citibank, which is the bank I use in the US. And they decided to leave. They sold all their customers and all their branches to another bank, a Brazilian bank.

Robert J. Marks: Boy, that's interesting.

Gregory Chaitin: It's worrisome. But on the other hand, Elon Musk and Stephen Wolfram make me think that the United States can still do basic innovation in spite of everything. Maybe the bureaucracy is not so bad. When I lived in France, people told me that to have a startup is much easier in the United States than in France. The rules and regulations are very tough for a number of reasons that I won't go into, and that I actually don't remember very well. So that's probably why Elon Musk wanted to come to the US. He came from South Africa.

Robert J. Marks: And now he's moved from California to my backyard in Texas.

Gregory Chaitin: Probably because California... all of the aerospace companies have left. Aerospace manufacturing is all gone from California. Too much bureaucracy. Texas is a freer place.

Robert J. Marks: Yep, so far. I hope we can keep it that way.

Gregory Chaitin: Let's hope so. So Elon built his spaceport in Southern Texas. Fortunately there's still Texas. Texas could be a separate country.

Robert J. Marks: It can. And there's rumors that when we joined the union, we were a separate country.

Gregory Chaitin: You spent ten years as a separate nation then, didn't you?

Robert J. Marks: Yes. The Republic of Texas. In fact I work at a place, Baylor University that was founded when Texas was a Republic.

Gregory Chaitin: Well, I worry about creativity. I have a chapter on that in my book, *Proving Darwin*. And I make a remark that from the point of view of creativity, I think that the best thing to do would be to split up the European community in separate countries and split up the US in separate states, because that would give more freedom of action to creative people instead of having a central bureaucracy.

Robert J. Marks: And it would set the free enterprise system into effect.

Gregory Chaitin: Once I asked a Greek, "How come ancient Greece was so innovative?" And he told me, "Well, the ancient Greeks asked that question themselves," and one answer, I don't know where the answer is, he didn't tell me that. But he said, because Greece was divided into separate city states. And actually it wasn't just Athens, the talented people would come from other city states and they would go to Athens. Whereas Egypt was very uncreative, and why was that? That was because it was flat and they weren't split up in separate islands or on land divided by volcanoes as Greece is or by mountains. So a central government was able to control all of Egypt and as a result, Egypt wasn't very innovative, ancient Egypt. And the crazy Greeks were always fighting each other and always with these separate little nations, the city states.

Robert J. Marks: You wrote a book for Springer in 1999, it was called *The Unknowable*. And also there was a tribute book to you, *Unravelling Complexity: The Life and Work of Gregory Chaitin*, and you were solicited to be an author of one of the chapters, it was the second chapter and it was entitled, "Unknowability in Mathematics, Biology, and Physics". This is a big deal. What is unknowability from your perspective?

Gregory Chaitin: Well, so I have a number I was proud of discovering called the halting probability Omega. You can define it mathematically very simply. Well, maybe not that simply; that in general you can't prove that programs are elegant is a simpler incompleteness result. The fact is that the sequence of base two bits of the numerical value of the halting probability Omega is maximally unstructured and maximally unknowable. It's irreducible mathematical information. Actually you have to go through a course on algorithmic information theory to see that, even though the ideas are simple. But the question is, does the number Omega exist? It exists in the mind of God. To an infinite mind, it is knowable. You could know each bit, but we're not infinite minds. You could argue that Omega doesn't even exist, that it's a fantasy. Nevertheless, Omega doesn't involve very sophisticated math, there's much wilder mathematics.

Gregory Chaitin: But the numerical value of this number Omega is maximally unstructured and maximally unknowable. We cannot know the value of each individual bit in the numerical value of this number. But you can counter attack saying this is a fantasy. The real number to an engineer is a fantasy. A real number has infinite precision. You don't work with infinite precision. We're happy to get a few decimal digits, different problems need more precision. But infinite precision is nowhere to be found except in the imagination of mathematicians who talk about a real number, which is determining the position of a point with infinite precision. But then the mathematics is simpler, it makes for beautiful mathematics. And a lot of physics is based on partial differential equations and mathematics, and depends on the fantasy of infinite precision real numbers. So in a sense, it's justified by its practical applications, but nowadays not so much because people don't solve equations that much anymore to do engineering, they use computer simulations.

Robert J. Marks: Or they go to Mathematica.

Gregory Chaitin: Which is using finite precision mathematics, not infinite precision real numbers like you do when you have equations and you want to prove properties of the solutions of the equations. This is how things used to work. Or find an analytic solution to an equation. That only works for very simple systems that you can solve in closed form mathematically and that's disappearing into the past. People are more concerned now with calculating than they are with proving theorems. In that sense, I'm a dinosaur.

Robert J. Marks: Well, in fact, we have a name for them in engineering, we call them keyboard engineers. If they're presented with a problem, they don't go to the theory which gives you depth and insight into what's happening. They to go to a keyboard and try to work things out just to get a surface answer.

Gregory Chaitin: And that's good enough for most practical purposes, right?

Robert J. Marks: Yes.

Gregory Chaitin: Instead of messing around with very complicated equations and trying to find an analytic, a closed form solution, you simulate the system and see how it behaves. And you change the design a little bit to see if it behaves better, more like what you want. So that's a new paradigm that the computer enables us to use. You can argue that the Omega number doesn't exist because it allows an unlimited amount of time for calculations. Will a program halt if it's given an arbitrarily large amount of time? But computers will not last for an arbitrary amount of time. They'll break or the earth will freeze, or the sun will go nova. It's like talking about unicorns or flying horses.

Robert J. Marks: Let me tell you a pushback that I got from the idea of unknowability. I mentioned that I'm a big fan of your proof that elegant numbers are unknowable. It's very insightful. It's beautiful.

Gregory Chaitin: It's a very simple proof.

Robert J. Marks: It's a simple proof and according to Paul Argos, it would probably go into God's book as the simplest explanation of something.

Gregory Chaitin: Thank you. That's the nicest thing anyone ever said to me. Most people prefer the Omega number, it fascinates people. A religious person once in Vienna told me that he viewed Omega as a step closer to God because it shows in the mind of God there is a numerical value, but we can't get to it, right?

Robert J. Marks: Yes. Well, also there's your Omega, your work on the Omega number, which is referred to as Chaitin's number. That also belongs in God's book. It's also beautiful.

Gregory Chaitin: Thank you. Jack Schwartz though, he never liked my work on the Omega number. He liked the proof that you can't prove that a program is elegant.

Robert J. Marks: Oh, really?

Gregory Chaitin: Yeah. And I greatly admired him.

Robert J. Marks: And we're going to, in a subsequent podcast, get deeper into Chaitin's constant or what you call the Omega number. Here's the pushback that I got back from this, Greg, is that I mentioned your proof of elegance, that elegance was unknowable to somebody and they say, "Well, that doesn't mean it's unknowable, it means only that it's non-computable." And I guess my response was, "Non-computable, does that equate to unknowable?" I think that that's the assumption here. And what's your take on that?

Gregory Chaitin: Well, there's no program to calculate it. There's no way to prove it. So I don't know in what sense it could be knowable. Now for all practical purposes, you can determine for example, whether a program is elegant, if you run it for a lot of time. It's like the halting problem. If you limit the amount of time to some reasonable amount like one day of calculation, which is certainly a lot, you just run it for a day and you see whether it halts or not, and if it hasn't halted, you just assume it's never going to halt. For all practical purposes that may be a good answer. But if there is no time limit, then everything becomes a mathematical fantasy, you see. But mathematics deals with fantasies because they have clean, beautiful properties and you can prove theorems about them.

Gregory Chaitin: And mathematical fantasies often serve practical purposes in, for example, theoretical physics. So it's a complicated issue. Let me give another example of unknowability. If you toss a coin a lot of times, almost certainly the result is algorithmically unstructured or random, the sequence cannot be compressed into a smaller program. And you can even get estimates of how very probable this is, and it can be enormously high probability. So will you allow that as evidence? Toss a coin and get a lot of zeros and ones, do it a reasonable number of times N, not just three times. Most probably the resulting sequence will be very close to maximum program size complexity, which would be N bits. There won't be any programs substantially shorter than N and you can quantify that and say what the cutoff is.

Gregory Chaitin: So something is almost certainly unstructured, but you can never prove it from mathematical axioms that are less complicated than the number of bits of the sequence you're trying to show is random. And there's also the question of which axioms are you using. My belief is that pure mathematics should evolve, should be creative. You can add new principles. And in fact during my lifetime, I've seen a number of new principles added to pure mathematics. So these questions are all complicated to discuss like most philosophical question, on the one hand this and on the other hand that, and they're all good arguments and you pays your money and you takes your choice, right?

Robert J. Marks: Yes. So elegant programs, just to remind the listeners, an elegant program is the smallest program to achieve some objective. There exists the smallest program, so that is the elegant program. You can also think of it as a... I'd like to think of it in terms of images. If you have a big image, what is the most that you can compress that image? And that would be the elegant program for the image. Greg, I know that you've spent a lot of time with your proof that elegant programs are unknowable. Do you have a simple explanation that you can walk through that people of modest education could understand?

Gregory Chaitin: Well, I try.

Robert J. Marks: Let's give it a try.

Gregory Chaitin: It's in my experiment in autobiography that's included in the book *Unravelling Complexity*. By the way, there's another *festschrift* that they did for me when I was 60. *Unravelling Complexity* was intended to be for my 70th birthday, but it took a little longer. It got published when I was 73 or 72. So there are two *festschriften* actually. The experiment in autobiography is a short essay I wrote for *Unraveling Complexity*. In it I go through what I see as the essential ideas in the proof. I've often tried explaining the proof when I'm invited to give a talk at a university. And I have a funny test for whether I explained it well. When I explain it well, people laugh when they realize the paradox of how the proof works, people laugh. And when people just stare at me with glazed eyes or they don't laugh, that means I didn't explain it well. So let me see, I can try to give an even simpler explanation than is in the experiment in autobiography.

Robert J. Marks: Well, Greg now I'm paranoid. I don't know. I hope I laugh after your explanation, we'll see.

Gregory Chaitin: So here's the idea. Let's say you have a formalization of what provable means, provable from a fixed set of axioms and rules of inference. This is a formalization of axioms for mathematics. And Hilbert thought there would be one system that all mathematicians could agree on. And so once you've taken all the subjective element out, it's like a computer program. You just run through all possible proofs. You check which ones are correct. You filter out the correct proofs and that way you get all the theorems, all the theorems of mathematics it would be if Hilbert had come up with a formal axiomatic theory for all of math.

Gregory Chaitin: So now whatever the system you're looking at with whatever the axioms and rules of logic, it can be implemented as a computer program. From the proof checking algorithm that always gives an answer — the proof is correct, the proof is incorrect — you can go to an infinite runtime algorithm that generates all the theorems in order of the size of their proofs. That's just a small step. So you just look at the size in bits of either the proof checking algorithm or this infinite runtime algorithm. Actually, it's better to look at the algorithm that runs through and checks all possible proofs and gives you all the theorems. And you look at the size in bits of the program for doing this. This is the algorithmic information content or the complexity of that formal axiomatic system you're studying to see what it can achieve.

Robert J. Marks: If I remember right, you step through numbers one at a time and check if it's meaningful or not.

Gregory Chaitin: You check all possible proofs, one at a time.

Robert J. Marks: And those correspond to numbers, is that correct?

Gregory Chaitin: Well, they correspond to strings of characters in the alphabet. You can also just go through the tree of all possible proofs. That's another way to get all possible theorems with a one by one endless computation. So this thing will require a certain number of bits. That's sort of the bits of axioms you're using in this version of mathematics, which people thought they would have a definitive version of. So let's say the program that does this — either the proof checker or the one that runs through the tree of all possible proofs or the one that checks all possible proofs in size order and gives you all theorems — let's say this is N bits, whatever that is. So now you start running through all possible theorems until you find a proof that a program that is substantially more than N-bits long is elegant, which means it's the smallest program that calculates the output it does.

Gregory Chaitin: So you want to see if your formalization of mathematics and all its principles enables you to prove that a program is elegant that is larger than the software embodiment of those mathematical principles in that particular formal axiomatic system, its axioms and its logic, its rules of inference. You keep running through all the theorems, all possible proofs and all the theorems until you find a proof that a program is elegant that is substantially bigger in bits than the number of bits for the software implementation of this process to run through all possible proofs and get all the theorems. And then what you do is, you take that program, which is provably elegant and you run it, and then you see what its output is. And this is your output.

So what we have here is a process, a program that is basically the same number of bits in size as the number of bits of axioms in your mathematical theory.

Gregory Chaitin: And we're using this theory to attempt to prove that programs are elegant. And we keep looking through all the theorems until we find a proof that a string of zeros and ones — a finite string of zeros and ones that is substantially larger than the software implementation of your mathematical system — is elegant. So then you run this elegant program and its output is your output. This is a process you do that will come to an end and give you an output. Take a good look at this program that I've just described in words. It has a number of bits which is basically the number of bits for the software implementation of your axiomatic theory. And it's giving you an output, which is the output of the first provably elegant program that is substantially larger than the number of bits in your mathematical theory. Now this is impossible, because this output that you've gotten, supposedly is the output of a provably elegant program.

Gregory Chaitin: And that means that this is the smallest, most concise program that can calculate the output that it does. But you've calculated it with a substantially smaller number of bits because you found it by running through all possible theorems, all possible proofs in your mathematical theory. And by the construction of this paradox, you keep running this process until you prove that a program is elegant that has substantially more bits than the program that is the software implementation of your mathematical theory. So now, this has given you a smaller program than the supposedly elegant program to calculate this object. And that's impossible by the definition of elegance. So either you're proving false theorems, or you can never find this program. If you only prove true theorems, the elegant program for the thing you calculated is substantially larger than the number of bits in the program that found the proof that this program is elegant.

Gregory Chaitin: So you've actually compressed the output from this supposedly elegant program into a smaller program. And that's impossible by the definition of elegance. So if you assume that only elegant programs can be proven to be elegant, in other words that the theorems you're proving are all true, then you'll never find a proof that a program is elegant that has substantially more bits than the software implementation of your mathematical theory. The only way you can avoid the contradiction is if this process never finds a proof that this program that is substantially larger than your software implementation of your mathematical theory is elegant. So in other words, any mathematical theory can only prove that finitely many programs are elegant, they have to be smaller in size than the software implementation of the mathematical theory. But there are an infinite number of elegant programs, because...

Robert J. Marks: They get bigger and bigger and bigger.

Gregory Chaitin: Because for any programming task, there is a most concise program for it. The problem is you're not going to be able to prove that you've got it if the number of bits in your program is larger than the number of bits in your mathematical theory. I don't know if this was understandable or not. It depends on the notion of a completely formalized objective, not subjective version of mathematics. Now how seriously should we take this? In fact, Gödel thinks that mathematicians are not limited by his incompleteness theorem because they can directly intuit facts from the Platonic world of ideas.

Gregory Chaitin: This incompleteness result that I just explained only applies to totally formalized computerized mathematical theories, but Godel doesn't think it applies to human mathematicians. Unfortunately, to be able to say what you can prove and show that there are limitations, you have to give a very precise definition of the methods you're allowing in the proofs. And once you do that you're in trouble because if it's N bits of methods that you're allowing for mathematical proofs, then elegant programs that are more than N-bits long will not be provably elegant. Does that sound more understandable?

Robert J. Marks: So it's a proof by contradiction. You assume that the elegant program detector was algorithmic, and then you showed that there was a contradiction in your assumption and therefore it can't exist.

Gregory Chaitin: Nobody laughed, so I guess I fumbled the ball.

Robert J. Marks: Nobody laughed. What was the joke, did I miss it?

Gregory Chaitin: Well, it's the contradiction.

Robert J. Marks: Oh, the contradiction. Maybe it was because of my familiarity with the proof.

Gregory Chaitin: I explained it badly.

Robert J. Marks: No, I don't think so at all. I think that in my class, when I explained proof by contradiction, let me tell you my favorite example. It's the proof that all positive integers are interesting.

Gregory Chaitin: That's related to all of this stuff.

Robert J. Marks: It is. And so you've heard about this. So the idea is you assume the opposite. Assuming that some numbers are uninteresting, then there is a smallest non-interesting number, but "Hey, that's interesting." So that's the proof by contradiction.

Gregory Chaitin: That's similar because interesting number, if you want to define it carefully mathematically, is one for which there's a program to calculate it that's smaller than it is. So an uninteresting number would be one whose numerical value is irreducible. And that's the basic idea of my proof. You're right, that's a good way to explain this. That's a very good explanation because then the next step from that to get to my incompleteness theorem is to say, well, what does interesting mean?

Gregory Chaitin: And one good definition of interesting is an interesting number is one that stands out because there is a more concise definition of it, or more precisely a program that is substantially smaller than its numerical value that calculates it, That's how it stands out from the run of the mill numbers. And the run of the mill numbers are ones whose numerical value is incompressible or an irreducible string of bits. So you can go step by step from that paradox about the smallest uninteresting number, which is *ipso facto* interesting, to a proof of an incompleteness result very similar to mine.

Robert J. Marks: Very interesting. New topic, I wanted to talk to you about... we're talking just in general about the unknowable. Roger Penrose, he recently won a Nobel Prize for his work with Stephen Hawking in black hole theory. Wrote a book called *The Emperor's New Mind*, and he had a follow up, which is the *Shadows of the Mind* or something like that. But in the book, he says that creativity is non-computable. He uses your work along with Turing's work and Gödel's work to make an argument that creativity is non-computable, and therefore is something which if we're to pursue it by artificial intelligence, something that will never be done. Now he refers to quantum as possibly the only non algorithmic thing that occurs in nature. So he says all of this stuff must be due to quantum collapse in cellular microtubules in our brain or something like that. And I'm wondering if you have any thoughts on whether creativity is computable or not computable, or do we know yet?

Gregory Chaitin: I do have thoughts on this. I'd like to make one or two remarks. I met Penrose in Cambridge at a meeting, we were both speakers. And I went up to him and I said, "The reason you think that a machine can't equal human intelligence, is because you believe that we have a divine spark and computers don't have a divine spark." And he answered a trifle annoyed, I think, "Not at all."

Robert J. Marks: Well, he did revert to a materialistic solution, which is quantum mechanical. So yes.

Gregory Chaitin: I've been worried a lot about creativity lately when I started working on biological creativity and I connected it with mathematical creativity. There is this paradox, just like the first uninteresting number is *ipso facto* interesting, there's a problem with creativity, with having a algorithm for creativity, a computer program for creativity. The problem is that if you know how to do something, *ipso facto* it's not creative anymore.

Robert J. Marks: And that's the problem with identifying creativity. One can have a creative spark and you explain it to somebody, they sit there and rub their chin and say, "Well, that's obvious."

Gregory Chaitin: Well, creativity is what we don't know how to do. And so it looks like it's a hard thing to program because if we try to program creativity, well that just becomes something mechanical. And the frontier between what's creative and what isn't just moves a little forward. It's a problem. There could be a mathematical... I think that my attempt to find a little toy model of biological evolution — this is a controversial — is a first step in the direction of a mathematical theory of creativity. I believe that Gödel's incompleteness theorem and Turing's work on the uncomputability of the halting problem are baby steps, well, they're big, big baby steps in the direction of a theory of creativity. That's normally not how you view them, but I feel they feed into my little attempts to look at biological creativity using a painfully simplified toy model. So there is a paradox about creativity.

Gregory Chaitin: Now you could have a mathematical theory of creativity that enables you to prove theorems about creativity, but is not implemented in software. That doesn't give you an algorithm for being creative. Because if it's an algorithm it's not creative. But you might be able to prove theorems about creativity. You see, I can prove theorems that most numbers are random, or unstructured, but I can't produce individual examples that I'm certain are. So it might be that you could prove theorems about creativity, but the theory wouldn't give you a formula, a recipe for being creative because once it does that then it's not creative. You see, there is this paradox.

Robert J. Marks: And also those theorems that you're talking about are meta. You're using creativity to write theorems about creativity.

Gregory Chaitin: Of course, absolutely.

Robert J. Marks: And one of the important things is to define creativity. I know somebody that knows you, I'm not sure if you know him, Selmer Bringsjord, who said he met you I believe at a recent meeting, but he has something called the Lovelace test, which is a lot better than the Turing test. He says that a computer program will be creative if, and only if that computer program does something which is outside of the intent or explanation of the programmer. And I think that that's a very... the Lovelace test is one that I believe hasn't been passed yet. So it's going to be interesting to see if future AIs can do anything creative.

Gregory Chaitin: Well, I think I met this gentleman in Thessaloniki.

Robert J. Marks: Yes. I believe that was where he mentioned that he met you.

Gregory Chaitin: Well, Turing says he'll believe that a computer is intelligent if the computer will punish him for saying that computers aren't intelligent.

Robert J. Marks: Now see, that's funny. What did he mean by punish you? Come up and whack you across the head or what do you mean by that?

Gregory Chaitin: I guess Turing is referring to a notion of truth based on political considerations. People will say something's true if the society will fire you from your job if you disagree.

Robert J. Marks: The other thing you mentioned in your book is Tononi's Phi function model of consciousness, which I must admit, I don't totally understand.

Gregory Chaitin: Me neither. It's complicated.

Robert J. Marks: Well, that's good. It is complicated. And Christof Koch, in one of his presentations about the theory, he got the people in Silicon Valley mad because, this was a report that I got from somebody who attended Koch's lecture, they were mad that this was possibly non-computable, at least with the resources that we have now or in the immediate future. So I don't know, it seems to me that there's... who was it? It was Stephen Hawking who says, "Nothing is ever proven in physics, you just accumulate evidence." And so I think evidence is accumulating in so far as the non-computability of some human attributes, at least that's my personal take.

Gregory Chaitin: I had something I wanted to say.

Robert J. Marks: About Tononi?

Gregory Chaitin: Yeah. I actually prefer... Calculating Phi is complicated and you need to do an immense amount of computing. You have to look at all possible partitions of a physical system and calculate certain mutual informations for... it's a horrendous, exponential growth computation. I know Phi is fashionable now, but I prefer the original approach in Chalmers' book, I believe 1996 was it?, *The Conscious Mind*. He has an idea similar to Leibniz's *Monadology*, it's Chalmers' version of panpsychism.

Robert J. Marks: Oh yes.

Gregory Chaitin: Everything has some degree of consciousness. It may be greater, it may be smaller. The maximum monad corresponds to God, whose consciousness is the largest possible consciousness of everything. A rock doesn't have that much consciousness. But Chalmers believes a physical system has N bits of consciousness if it has N bits of memory and processes these N bits. So that would mean that an on-off light switch would have one bit of consciousness and a human being would have a lot of bits of consciousness. It's hard to have a cutoff, for example, if humans are conscious and the people who love dogs are certain that dogs are conscious, is there a sudden place where consciousness blanks off as you go to more primitive life forms, bacteria, viruses, light switches?

Gregory Chaitin: So it looks a little implausible from a philosophical point of view. It seems more likely that it'll just be gradually less and less conscious. And you can go in the other direction, you can have a corporation, does that have consciousness? Does the internet have consciousness? Does the whole universe have consciousness? Which presumably would be God. Actually, my latest and hopefully not last paper is on consciousness, and it's just being published in a book in honor of one of my distinguished colleagues here in Brazil, Francisco Antonio Doria. It's "Consciousness and Information, Classical, Quantum, or Algorithmic?". Because Chalmers...

PART 3 OF 4 ENDS [01:45:04]

Gregory Chaitin: ... because Chalmers didn't know that there are three definitions of information. There's Shannon information entropy, there's quantum information theory, which maybe didn't exist in 1996 or was very incipient, and now it's a big, fashionable topic. And there's algorithmic information.

Robert J. Marks: It's interesting. I've always looked at panpsychism as a weird philosophy. I'm wondering if there's any way that it can be tested. I doubt it. It's going to be interesting to see if it can. But the posit is that consciousness is part of the universe just like mass and energy and all of the other stuff.

Gregory Chaitin: Well, there are idealistic philosophies which say that the universe is spirit and matter is an illusion. That's related to an idea that I've been backing, which is that the universe is made from information, that that's the ontological basis. The normal view, if you dabble in metaphysics, is that the universe is made from mathematics. That's the Pythagorean idea that God is a mathematician. I prefer to say all is algorithm, God is a computer programmer. There's a book by an Italian theologian, a priest, on this subject called *Bit Bang. La nascita della filosofia digitale.* It's a wonderful book, but unfortunately it's only available in Italian. So saying that the universe is built out of information is like saying that the universe is built out of spirit or the universe is in the mind of God. It's not a material substance.

Gregory Chaitin: And the new version that physicists love is to say that the universe is built out of quantum information. They want to try to get everything out of quantum information, including having spacetime emerge from entanglement between qubits, for example. That's a fashionable topic. But in a way here we're looking at an old idea, which is that the universe is in the mind of God or the universe is spirit not matter. This is idealism as opposed to materialism, which is why a theologian was interested and put together a book surveying all of this work. Unfortunately, I don't think anybody has translated his survey. Maybe Google Translate could do a decent job.

Robert J. Marks: Yeah, maybe. I think that still has a long way to go. Chaitin's number, one single number between zero and one allows under certain conditions, solution of every open problem in mathematics that can be disproved by a single counter-example. Many of these problems have large cash prizes for their solution. And we'll find out why, even though Chaitin's number exists, many of these cash prizes remain unclaimed. I want to clear up something first of all. Stanford's Thomas Cover and Joy Thomas wrote a book that I used as a textbook for the graduate course in information theory called *Elements of Information Theory.* They refer to Chaitin's "magical mystery number Omega." And this is in a very thick scholarly book, they call your number mystical and magical. Now, of course in your writings, you do not refer to this as Chaitin's number. You refer to it as a capital Omega. Let me clear up something. There are people like Cover and Thomas that refer to this as Chaitin's number, whereas if you look at Wikipedia, they call it Chaitin's constant. Which one is correct, Chaitin's number or Chaitin's constant?

Gregory Chaitin: It's a little worse than that.

Robert J. Marks: It's worse than that. Okay.

Gregory Chaitin: Because you see, there isn't one halting probability Omega. It depends on the computer programming language.

Robert J. Marks: Exactly. So the Chaitin's number varies in accordance to the computer program you're using.

Gregory Chaitin: Right.

Robert J. Marks: Therefore calling it a constant doesn't seem to be...

Gregory Chaitin: No.

Robert J. Marks: ...appropriate.

Gregory Chaitin: But its bizarre or fascinating properties don't vary as long as the computer programming language you pick is a universal Turing machine which allows very concise programs, a general purpose computer that allows programs that are as concise as possible. So you have to be a little careful which programming language you use, but there are an infinite number of programming languages that will give you a bonafide halting probability Omega with all the madness in it or all the fun in it. So I think Wikipedia refers to Chaitin's procedure or something. Anyway, this is a quibble, I think. In fact, in my books, I actually settle the programming language. I use a version of LISP to write an interpreter for the programming language that is used to define the halting probability. And once you do this, the halting probability is a definite number with a definite numerical value that is maximally unknowable.

Gregory Chaitin: By the way, in my first paper, I used a low case Omega ω. My first paper on Omega was published in the *Journal of the ACM* in 1975. I already mentioned that paper. I used a low case Omega. Omega is the last letter of the Greek alphabet. At churches in Europe, you'll see alpha such a year, omega such a year, that's when they began building it, that's when they finished building it. And there's omega and omicron. Omicron is little omega and omega is big omega.

Gregory Chaitin: Anyway, Robert Solovay, who was working at IBM when I wrote the paper where Omega appears for the first time, he said, "Oh no, you don't want to use low case Omega because in set theory, low case Omega stands for the set of natural numbers, $\omega = \{0, 1, 2, 3, 4, 5 \ldots\}$. So use a big Omega Ω." So henceforth, I took Bob's advice. He's one of the best set theorists on the planet. And it became capital Omega. And I was sort of surprised that some people referred to it as Chaitin's number. I've even seen it in lists of important mathematical constants.

Robert J. Marks: Even though it's really not a constant, it's a number.

Gregory Chaitin: Though it's not a constant, unless you fix the programming language which I actually do in one of my books. For other constants people give the numerical value up to a certain level of approximation, and mine is in some of those lists simply because you can't do that. Well, maybe you can get a few bits of the numerical value, but at some point it becomes unknowable.

Robert J. Marks: Has anybody computed the first few bits of Omega?

Gregory Chaitin: Yeah. My colleague, Cris Calude and another professor at the University of Auckland have a paper, I think called "Computing the Bits of Omega" or something like that. They pick a different computer than I do, but it's a good computer. It's a universal Turing machine and it allows for the most concise possible programs. And they're able to determine, I don't remember if it was 40 bits of the number, and those bits are on one side of the Leibniz medallion that Stephen Wolfram gave me as a present when I was 60, in 2007. On the other side, it has a medal that Leibniz wanted his Duke to coin in silver, that celebrates binary arithmetic. Leibniz came up with binary arithmetic a long time ago. And he came up with the idea that the whole universe might be made out of information because that side of the medallion shows some addition and some multiplication and it says *imago mundi*, that's Latin for image of the world, and in the German version it's *Bild die Schöpfung*, which means picture of the creation instead of image of the world.

Gregory Chaitin: I think Leibniz intuited that it might be possible to make everything from zeros and ones. That's digital philosophy.

Robert J. Marks: Digital, okay.

Gregory Chaitin: Digital philosophy which is a philosophical idea that I like to toy with. And the other side of the Leibniz medallion that Stephen Wolfram was kind enough to give me as a present for my 60th birthday has Omega, big Omega, on it, and it has some bits, the bits of the Omega number that Cris Calude and his colleague were able to calculate and prove were correct. So there's a contradiction because in Latin the medallion says the Omega number is something that's unattainable, but at the bottom it gives some of the initial bits of the numerical value of one Omega number. So that's a nice paradox, which is always stimulating.

Robert J. Marks: Well, here's a question that I have. I know that Omega or Chaitin's number, the foundation is the Turing halting problem, which says that there can be no program written to analyze generally another arbitrary program to say whether it halts or it runs forever. Wouldn't you have to, in a way, solve the halting problem for smaller programs in order to get Chaitin's number?

Gregory Chaitin: Yeah, well, if you had the first N bits in base two of the numerical value of the halting probability Omega, from that, it's very straightforward, but very time consuming to solve the halting problem for all programs up to N bits in size. So in a way, the halting probability Omega is a very compressed form of answers to individual cases of the halting problem. Knowing N bits of the value of the halting probability tells you for two to the N individual programs, all the programs up to N bits in size, whether each one halts or not, as it's not too hard to see.

Gregory Chaitin: One way to put it, from an engineering point of view, is that Omega is the maximal compression of the answers to individual cases of the halting problem.

Robert J. Marks: Yes.

Gregory Chaitin: It's an irredundant representation. It's provably the best possible compression of all the answers for individual cases of the halting problem, because solving the halting problem for all programs up to N bits in size cannot be done in less than N bits, otherwise you get a paradox, and Omega does it in precisely N bits.

Gregory Chaitin: So that is the best possible compression. It's like the crystallized essence of answers to the halting problem, the best possible irredundant compression of all the answers.

Robert J. Marks: And even though a few bits have been computed of Chaitin's number, it is advertised as unknowable.

Gregory Chaitin: Yeah, because you can show that an N-bit program can't calculate more than the first N bits and an N-bit mathematical theory can't enable you to prove what more than the first N bits are. So it's logically and computationally unknowable. If the tool you're using has N bits, you're going to be able to get at most N bits of the numerical value. And after that, all the remaining infinite number of bits look just totally random and totally unstructured and you'll never know them.

Robert J. Marks: That's fascinating.

Gregory Chaitin: So it's fun. I'm surprised that it became such a hit.

Robert J. Marks: Well, but think about that, when you tell somebody that there are all of these open problems in mathematics that require a single counter-example, Goldbach's conjecture, the Riemann hypothesis —these are all problems with big prizes, big monetary prizes — and you have postulated in theory, a number, Chaitin's number, Omega, a single number between zero and one that solves all of these problems, at least in the philosophical realm. And that is astonishing, that is mind blowing. That is something more extraordinary, as I mentioned, I think in the first podcast, than any science fiction that I've ever read or watched.

Gregory Chaitin: Okay, well, that's great. I hope it will inspire some young people. I've worked as an engineer. Elon Musk is my hero, but pure mathematics is also one of my loves and maybe Omega will inspire some young people to become mathematicians, pure mathematicians. And it may also help to give people an interest in philosophical questions instead of practical questions.

Robert J. Marks: Yes.

Gregory Chaitin: There are always going to be a few of us who think... There are a lot of us who like to do things that are practical, and that's part of my personality too, but there's also in me, a side that likes beautiful mathematical arguments and the fantasy world of pure mathematics, which has this strange number glowing there, the Omega number. And we do need some people who do pure mathematics with no applications, maybe not for 100 years.

Gregory Chaitin: And we also do need some people who think philosophically, and I think Omega is a stimulus. So it's nice to have this number. I didn't expect it to catch on the way it did. I wouldn't say it's complicated because I invented it, but it takes a mathematical theory that is non-trivial to understand exactly how you define the number and why it has the properties it does. I thought the business that you can't prove that a program is elegant would catch on more, but people are interested in philosophical questions — even though Omega is sort of a unicorn or a flying horse, it's a mathematical fantasy in the Platonic world of ideas — it has caught on, which shows that we all have a taste for that kind of question too, which is good. The human spirit is capable of doing both things, practical things and impractical things which may have practical consequences later.

Robert J. Marks: I think the poster child for that is encryption involving large prime numbers and their factorization, which is used in most of the encryption that's used today. That was a mathematical theorem that laid around for a long time before somebody found out that, "Hey, we could use this for encryption purposes."

Gregory Chaitin: Yeah, who would've thought that could happen? G. H. Hardy loved number theory, because he said it's purest of the pure, pure mathematics. He has a book that was written during the Second World War, I think. And he'd seen the First World War and he hated things with practical applications because he said they would be used for carnage, for slaughter. So he said, at least there's one subject which will forever be as pure as the driven snow, because it's totally impractical, which is number theory. So now it's used for cryptography for sending military messages, probably. I don't know.

Robert J. Marks: Oh, absolutely.

Gregory Chaitin: And it's used also for banks, financial transactions. So Hardy would probably be very disappointed, but historically it's fun. The same thing happens with algorithmic information theory. At least for me, it was purely philosophical. I was interested in incompleteness. Solomonoff was interested in more practical stuff, but it looked difficult to handle applications.

Gregory Chaitin: Algorithmic information theory goes back to the '60s. So where are we now? It's 60 years later, and Hector Zenil and his collaborators are using practical approximations to this ideal theory for practical applications. Oh, here is the biggest joke of all. The halting probability Omega is totally unknowable. It's uncomputable, but you can calculate it in the limit from below. You look at more and more programs, you see which ones halt, and that way, the halting probability keeps going up. Your estimate keeps going up. It's very, very slow this process, but in the limit of infinite time, you can calculate Omega in the limit from below. Well, the joke is that Hector Zenil has proposed using this as a new cryptocurrency that he calls Automacoin. And this is a serious proposal because he says that Bitcoin, which is the most popular cryptocurrency, uses an immense amount of computing power, a really scary amount of computing power that didn't even exist before. But it's all going to waste. It's not a terribly useful computation except for financial transactions.

Gregory Chaitin: But what Hector Zenil has basically proposed is to calculate the bits of Omega in the limit from below.

Robert J. Marks: Oh, my goodness. What an idea!

Gregory Chaitin: And this gives you a cryptocurrency where what you calculate is very, very useful. It's useful in the way that Marvin Minsky pointed out: it tells you the best theories for things, the most concise programs for things. And that can be used for making predictions.

Robert J. Marks: Well, one of the interesting things about this talking to George Gilder, who is kind of an economics guru and a forecaster of the future, he says one of the problems with Bitcoin is the amount of value that you can get out of it is fixed. So there's only a certain amount of gold that can be mined from Bitcoin software. If we use the Omega number as a basis for a cryptocurrency, there would be no limit, would there? It would just get harder and harder and harder to mine the gold.

Gregory Chaitin: Right. The further you go, the harder the computations, but the results of those computations are actually useful. Whereas in Bitcoin, you're spending a very, very large amount of computing power mining Bitcoins, and...

Robert J. Marks: Yes.

Gregory Chaitin: ...it's only useful for Bitcoins. It's not useful for anything else. So Hector thinks that's a terrible waste. I'm not terribly interested in practical applications, not when I'm thinking about mathematics and Omega and such, but the fact that it's proposed as a cryptocurrency, I think is fantastic. It's just a lot of fun. Who would've guessed?

Robert J. Marks: It is really interesting.

Gregory Chaitin: But you had to wait 60 years for this. When I was publishing these papers, if I had to show there were practical applications, I wouldn't have been able to publish that research. 60 years later, this is a practical research program based on the Omega number.

Robert J. Marks: Boy, that is just a fascinating, fascinating idea. One of the, I think the poster problem for the Turing halting problem is Goldbach's conjecture which says that every even number can be expressed as the sum of two primes. And if one had a halting problem algorithm or a halting problem Oracle, if you will, you could settle Goldbach's conjecture very easily by looking for a single counter-example or showing that no counter-example exists. Do you ever think that Chaitin's number can be calculated to a precision which would allow for the proof or disproof of something like Goldbach's conjecture, which is a relatively short program?

Gregory Chaitin: Yeah. It's relatively short as computer programs go, but there are a lot of programs up to that size. That grows exponentially. So the calculation gets quite horrendous, and the algorithm that extracts, that tells you, given the first N bits of Omega, that tells you for each of the programs up to N bits in size, which one halts and which one doesn't, if you do the obvious algorithm, its runtime is worse than super exponential. It grows as the Busy Beaver function of N.

Robert J. Marks: Oh gosh.

Gregory Chaitin: It looks tough. By the way, there's another interesting example of a famous mathematical problem called the twin prime conjecture that says that there are infinitely many prime numbers that are two consecutive odd numbers.

Robert J. Marks: Yes.

Gregory Chaitin: And that certainly seems to be the case. In fact, there are very good estimates that are empirically validated formulas that tell you the distribution of twin primes, how many there are. It can't be proven, but there's a formula which gets more and more accurate the more twin primes you calculate. So it certainly looks like there're infinitely many, and we even know the distribution, at least in the sense that an empirical scientist knows anything. But the question of whether there are infinitely many twin primes is not equivalent to a halting problem.

Robert J. Marks: Yes, it can't be solved with a Turing Oracle.

Gregory Chaitin: Yeah, there's no finite counter-example. So you can't have a program searching for a counter-example. And then if the program doesn't halt, you know there is no counter-example. Omega doesn't solve all problems. Now there are extended versions of Omega. There's a hierarchy of Omegas that look at more and more abstract mathematical questions. So knowing the bits of Omega wouldn't allow you to solve the twin prime conjecture positively or negatively, but there's a thing called the jump of the Omega number that sort of would. So you have Omega, Omega prime, Omega double prime...

Robert J. Marks: So Omega prime assumes that you have a halting Oracle.

Gregory Chaitin: Exactly. That's correct.

Robert J. Marks: And so you have this Omega prime that includes a halting Oracle, but...

Gregory Chaitin: That's right.

Robert J. Marks: ...if you have a halting Oracle, then you have to have a meta halting Oracle that looks at regular computer programs with the regular halting Oracle. So the Turing halting problem becomes more and more problematic, but in each case, you have more and more sophisticated things that you can do with Omega which is just astonishing.

Gregory Chaitin: But you would need an Oracle to do those things.

Robert J. Marks: Yes. And those are probably not computable. I'm wondering if there's any way to get a handle on what Omega prime is.

Gregory Chaitin: Yeah. Well, a colleague of mine in Buenos Aires, Veronica Becher and I mostly at her initiative worked on Omega prime and even on Omega double prime. Another way to define Omega prime is, it's not the probability that a program will halt, it's the probability that a program will produce a finite amount of output instead of producing an infinite amount of output. These are programs that never halt.

So the probability that such a program generated at random produces only a finite amount of output is Omega prime.

Gregory Chaitin: How about Omega double prime? Let's consider programs calculating only positive integers. Omega double prime is the probability that such a program produces all but a finite number of the positive integers. So these are programs that calculate only zero, one, two, three, four, five... And the question is the numbers that it doesn't calculate, will that be a finite number of numbers, or will it be an infinite number of numbers? That's Omega double prime. This was work mostly done by Veronica Becher and students of hers, and I participated a little bit in that work. And by now it's probably been generalized and extended in all directions like what happens in pure mathematics all the time.

Robert J. Marks: Well, that's the thing, Omega itself is mind blowing. But the fact that there's a regress into Omega prime and Omega double prime is hyper mind blowing if you will. And it goes on and on, doesn't it?

Gregory Chaitin: Yeah. This hierarchy of things that are more and more unknowable, there's a normal computer, there's a computer that has an Oracle for the halting problem, then there's a computer that has an Oracle for a computer that has an Oracle for the halting problem. And it goes up and up.

Robert J. Marks: Yes.

Gregory Chaitin: It's a sort of a hierarchy like Cantor's theory of infinities. There's the infinity of the integers, the infinity of the reals. And for any infinity, there's a bigger infinity, which is the infinity of all subsets of the previous step. So you have a hierarchy and dare I say that this is the notion maybe that you can only approach God in the limit. I think Cantor was interested in this question.

Robert J. Marks: Well, in fact, I believe that Cantor attempted to make an audience with Pope Leo the 13th. I believe it was Pope Leo, the 13th, about his theory of transfinite numbers. He never got to the Pope, but he got to one of the Pope's subordinates because he thought about the theological implications of his theory of the infinite.

Gregory Chaitin: Absolutely. I regard Cantor's theory of infinities as mathematical theology. It's beautiful stuff. It's also paradoxical. And apparently, Cantor thought that God was speaking to him. He felt he was inspired. In a way, to anyone who is creative, it sounds like God is speaking to us. In mathematics that I would say is discovered not invented, you feel you're touching a reality beyond normal reality.

Robert J. Marks: Yes.

Gregory Chaitin: And this is in that BBC 90-minute documentary, *Dangerous Knowledge*. It talks about this...

Robert J. Marks: Okay, I'm not familiar with this. What is the name of the documentary?

Gregory Chaitin: *Dangerous Knowledge*.

Robert J. Marks: *Dangerous Knowledge*?

Gregory Chaitin: It's on Cantor, Boltzmann, Gödel and Turing. I think it's a wonderful movie. The gentleman who made it, David Malone, actually went out to the places where these four men worked. And also in some cases, to the cemeteries where they are buried. Maybe it's not obvious that when he talks about these thinkers, David's actually showing you where they worked and maybe where they're buried and things like that. It's like a mathematical travel documentary. It's quite a fun film.

Robert J. Marks: I've always found that interesting. We know what these people did, but not who they were. And I think visiting the grave sites kind of tells us who they were.

Gregory Chaitin: I visited the Gödel and von Neumann grave sites at Princeton.

Robert J. Marks: You did? Oh, they're buried at Princeton? I didn't know that.

Gregory Chaitin: They're buried in the cemetery that's a short walk from Princeton University.

Robert J. Marks: I see. Is Einstein buried there also, do you know?

Gregory Chaitin: No, Einstein insisted that he be cremated and the ashes be scattered at an undisclosed location.

Robert J. Marks: Really? Okay. I didn't know that.

Gregory Chaitin: He didn't want to leave a shrine. I also visited Gödel's home, which is near the cemetery. It's in a poor part of town. Von Neumann had a home that I never visited unfortunately. I only found out where it was. It's in the wealthy part of Princeton and he and Gödel are together in the same cemetery, rather close to each other. Anyway, I visited Gödel's home and the people who were renting it from the current owner let me in and they showed me things that were left there from when Gödel was there.

Gregory Chaitin: For example, he had soundproofing put in because noise distracted him enormously. And his wife had a shrine to the Virgin Mary in the backyard, which was still there. And in fact, their home was in the neighborhood which when they bought the home were Italian laborers, people who did construction work, I think. It was in a relatively modest part of town. Gödel didn't marry a professor's daughter, he married a woman he fell in love with from comparatively humble origins. And I think she felt more comfortable with Italian neighbors. Also it was a Catholic neighborhood, whereas Princeton of course is some form of high Protestantism, I believe, or was at that time.

Robert J. Marks: Yeah, well certainly Princeton was founded on the Protestant ethic and boasted of a number of evangelical presidents for a while.

Gregory Chaitin: Ah, that I didn't know.

Robert J. Marks: Well, that's true of a lot of the universities. They were founded based on Christianity, but they have strayed from that task as time goes on.

Gregory Chaitin: Two notable examples are Oxford and Cambridge. Oxford was for teaching priests, Catholic priests. And there was a schism, so a group went off and founded Cambridge. This was a long time ago, probably something like 500 years ago.

Robert J. Marks: That's fascinating. Well, Professor Chaitin, this has been a delight talking to you. We've been talking to Professor Gregory Chaitin today about a number named after him, and we also went into some other topics. Chaitin's number is referred to as Omega, that's what he calls it, and Cover and Thomas in their classic textbook *Elements of Information Theory* referred to it as Chaitin's magical mystical number Omega. We did not have the time to go into the details of Chaitin's number, but we will give some links to some of Professor Chaitin's lectures where he explains this incredible, mind blowing number. And so until next time on Mind Matters News, be of good cheer.

Announcer: This has been Mind Matters News with your host Robert J. Marks. Explore more at mindmatters.ai. That's mindmatters.ai. Mind Matters News is directed and edited by Austin Egbert. The opinions expressed on this program are solely those of the speakers. Mind Matters News is produced and copyrighted by the Walter

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