The Chaitin Interview V: Chaitin's Number

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Robert J. Marks:

The most mind-blowing number in computer science and mathematics is Chaitin's number. Today, we talk to the man himself, Gregory Chaitin, about Chaitin's number.

Announcer:

Welcome to Mind Matters News, where artificial and natural intelligence meet head on. Here's your host, Robert J. Marks.

Robert J. Marks:

Chaitin's number - one single number between zero and one - allows, under certain conditions, solution of every open problem in mathematics that can be disproved by a single counterexample. Many of these problems have large cash prizes for their solution. We're talking today about the inventor of Chaitin's number, Gregory Chaitin, and we'll find out why, even though Chaitin's number exists, many of these cash prizes remain unclaimed. Professor Chaitin, welcome.

Gregory Chaitin:

Pleasure to be here with you today, Bob.

Robert J. Marks:

Thank you. Thank you very much. I want to clear up something first of all. Stanford's Thomas Cover and Joy Thomas wrote a book that I used as a textbook for the graduate course and information theory called, *Elements of Information Theory*. They refer to Chaitin's magical mystery number, Omega. This is in a very thick, scholarly book. They call your number mystical and magical. Now, of course, in your writings, you do not refer to this as Chaitin's number. You refer to it as a capital Omega.

Robert J. Marks:

Let me clear up something. There are people like Cover and Thomas that refer to this as Chaitin's number whereas, if you look at Wikipedia, they call it, Chaitin's constant. Which one's correct, Chaitin's number or Chaitin's constant?

Gregory Chaitin:

It's a little worse than that.

Robert J. Marks:

It's worse than that? Okay.

Gregory Chaitin:

Because you see there isn't one halting probability Omega. It depends on the computer programming language.

Robert J. Marks:

Exactly. So the Chaitin's number varies in accordance to the computer program you're using. Therefore calling it a constant doesn't seem to be appropriate.

Gregory Chaitin:

No. But its bizarre or fascinating properties don't vary. As long as the computer programming language you pick is a kind of a universal Turing machine, which allows very concise programs. A general purpose computer, which allows the most possible concise programs. You have to be a little careful what programming language you use, but there are infinite number of programming languages which will give you a bonafide a halting probability Omega with all the madness in it or all the fun in it.

Gregory Chaitin:

I think Wikipedia refers to Chaitin's procedure or something. Anyway, this is a quibble I think. In fact, in my books, I actually settled the programming language. I use a version of LISP to write an interpreter for the programming language that I want to base the theory on. Once you do this, it's a definite number with a definite numerical value that is maximally unknowable. By the way, in my first paper, I used a low case omega. My first paper was published in the Journal of the ACM in 1975, I believe already in a previous interview, referred to the paper and its title.

Gregory Chaitin:

I use a low case omega. Omega is the last letter of the Greek alphabet. At a church in Europe, you'll see alpha such a year, omega such a year, that's when they began building it. That's when they finished building it. There's omega and omicron. Omicron is a little Oh and omega is a big Oh. Anyway, Robert Salovey who at that time was working at IBM when I wrote that paper where omega appears for the first time. He said, "Oh no, you don't want to use low case Omega because in set theory, low case omega stands for the set of natural numbers. Zero, one, two, three, four, five. Use a big Omega.

Gregory Chaitin:

Henceforth, I took Bob's advice. He's one of the best set theorists in the planet. It became capital Omega. I was sort of surprised that some people refer to it as Chaitin's number. I've even seen it in lists of important mathematical constants.

Robert J. Marks:

Even though it's really not a constant, it's a number.

Gregory Chaitin:

Well it's not a constant. Right, unless you fix the programming language, which I actually do in one of my books. Also, the other constants that they give, they give the numerical value up to a certain level of approximation. Mine is on that list simply because you can't give. Well, you can maybe get a few bits of the numerical value, but at some point it becomes unknowable.

Robert J. Marks:

Has anybody computed the first few bits of Omega?

Gregory Chaitin:

Yeah, my colleague Cris Calude, with another professor at the University of Auckland of a paper, I think, called Computing the Bits of Omega or something. They pick a different computer than I do, but it's a good computer. It's a universal Turing machine, and it allows for the most possible concise programs. They're able to determine, I don't remember, it was 40 bits of the number and on the Leibniz medal that Stephen Wolfram gave me when I was 60 in 2007, on one side, it has the face of a medallion that Leibniz wanted his Duke to coin in silver, which is about the binary arithmetic.

Gregory Chaitin:

Leibniz came up with binary arithmetic a long time ago. He came up with the idea that the whole universe might be made out of information because he shows some addition, some multiplication, and he says, Imago Mundi, that's Latin for "image of the world." In the German version, it's a *bildung der schöpfung* - something like that, I don't have much German - which also means "image of the creation" instead of "image of the world." I think Leibniz intuited that it might be possible to make everything from zeros and ones. That's digital philosophy.

Robert J. Marks:

Digital philosophy. Okay.

Gregory Chaitin:

Digital philosophy. Yeah, which is a philosophical idea that I like to toy with. The other side of the Leibniz medallion that Stephen Wolfram was kind enough to give me as a present for my 60th birthday has Omega, big Omega on it. It has some bits, the bits of the Omega number that Cris Calude and his colleague were able to calculate and prove correct. That's a contradiction because in Latin, it says, the Omega number is something that you can't calculate, but at the bottom he gives the sum of the numerical value of one Omega number is given. That's a nice paradox, which is always good.

Robert J. Marks:

Well, here's a question that I have. I know that Omega or Chaitin's number is based on, the foundation is the Turing halting problem, which says that there can be no program written to analyze generally another arbitrary program to say whether it halts or it runs forever. Wouldn't you have to, in a way, solve the halting problem for smaller programs in order to get Chaitin's number?

Gregory Chaitin:

Yeah. Well, if you had the first *n* bits in base 2 of the numerical value of a halting probability Omega from that, it's very straightforward, but very time consuming to solve the halting problem for all programs up to *n* bits in size. In a way, the halting probability Omega is a very compressed form of answers to individual cases of the halting problem. Knowing *n* bits of the value of the halting probability tells you for two to the *n* individual programs, all the programs up to *n* bits in size, whether each one halts or not, as it's easy to see. One way to put it, from an engineering point of view, it's the maximal compression of the answers to individual cases of the halting problem.

Robert J. Marks:

Yes.

Gregory Chaitin:

It's an irredundant representation. It's provably the best possible compression of all the answers of bits of the halting problem, because you can... To solve the halting problem for all programs up to *n* bits in size cannot be done in less than *n* bits so you get a paradox. Omega does it in precisely *n* bits. That is the best possible compression. It's like a crystallized essence of answers to the halting problem, the best possible irredundant compression of all the answers.

Robert J. Marks:

Even though a few bits have been computed of Chaitin's number, it is advertised as unknowable.

Gregory Chaitin:

Yeah, because you can show that an *n* bit program can't calculate more than the first *n* bits. An *n* bit mathematical theory can't enable you to prove what more than the first *n* bits are. It's logically and computationally unknowable. If the tool you're using has *n* bits, you're only going to be able to get at most *n* bits of the numerical value. After that, all the rest of the infinite number of remaining bits look just totally random and totally unstructured and you'll never know them.

Robert J. Marks:

That's fascinating.

Gregory Chaitin:

It's fun. I'm surprised that it became such a hit in a way.

Robert J. Marks:

Well, but think about that, when you tell somebody that there are all of these open problems in mathematics that require a single counterexample, a Goldbach's conjecture, Collatz conjecture, Legendre’s conjecture. These are all problems with big prizes. Big monetary prizes. You have postulated, in theory, a number - Chaitin's number, Omega - a single number between zero and one that solves all of these problems, at least in a philosophical realm. That is astonishing. That is mind-blowing. That is something more extraordinary, as I mentioned, I think of the first podcast of any science fiction that I've ever read or watched.

Gregory Chaitin:

Okay. Well, that's great. I hope it'll inspire young people to... Engineering is great, wonderful. I've worked as an engineer. Elon Musk is my hero. But pure mathematics is also one of my loves and maybe Omega will inspire some young people to become mathematicians, pure mathematicians. It may also give people an interest in philosophical questions instead of practical questions.

Robert J. Marks:

Yes.

Gregory Chaitin:

They're always going to be a few of us who think. There are a lot of us who like to do things and practical things. That's part of my personality too, but there's also, in me, a side that likes beautiful mathematical arguments and the fantasy world of pure mathematics, which has this strange number glowing there, the Omega number. We do need some people who do pure mathematics with no applications, at least maybe not for a hundred years. We also do need some people who think philosophically, and I think Omega is a stimulus.

Gregory Chaitin:

It's nice to have this number. I didn't expect it to catch on the way it did. It takes, I wouldn't say complicated because I invented it, but it takes a mathematical theory that is non-trivial, to understand exactly how you define the number and why it has the properties it does. I thought the business that you can't prove that a program is elegant would catch on more, but people are interested in philosophical questions. Even though Omega is sort of a unicorn or a flying horse, it's a mathematical fantasy in the platonic world of ideas, it's caught on, which shows that we all have a taste for that kind of question too, which is good. The human spirit is capable of doing both things. Practical things and impractical things which may have practical consequences later.

Robert J. Marks:

Yeah. I think the poster child for that is encryption involving large prime numbers and their factorization, which is used in most of the encryption that's used today. That was a mathematical theorem that laid around for a long time before somebody found out that, "Hey, we could use this for encryption purposes."

Gregory Chaitin:

Yeah. Who would have thought that - G. H. Hardy loved number theory because he said it's purest of the pure, pure mathematics. He has a book that was written during the Second World War, I think. He'd seen the First World War and he hated things with practical applications because he said they would be used for carnage, for slaughter. He said, at least there's one subject which will forever be as pure as the driven snow, because it's totally impractical, which is number theory. Now it's used for cryptography, for sending military messages probably. I don't know.

Robert J. Marks:

Absolutely. Yes.

Gregory Chaitin:

It's used also for banks, financial transactions. Hardy would probably be very disappointed, but historically, it's fun. The same thing happens with algorithmic information theory. At least for me, it was purely philosophical. I was interested in incompleteness. Solomonoff was interested in more practical stuff, but it looked very difficult. And now what is it? Algorithmic information theory goes back to the 60s. What are we now?

Gregory Chaitin:

It's 60 years later, Hector Zenil and his collaborators are using practical approximations to this ideal theory and for practical applications. Oh, this is the biggest joke of all. The halting probability Omega is totally unknowable. It’s uncomputable, but you can calculate it in the limit from below. You look at more and more programs, you see which ones halt and that way, the halting probability keeps going up, your estimate keeps going up. But it’s very, very slow this process. But in the limit of infinite time, you can calculate it in the limit from below.

Gregory Chaitin:

Well, the joke is that Hector Zenil has proposed using this as a new cryptocurrency that he calls Automacoin. This is a serious proposal because he says that Bitcoin, which is the most popular cryptocurrency, uses an immense amount of computing power. Really scary amount of computing power that didn't even exist before. But it's all going for this. It's not terribly useful computation except for this financial transactions. But what Hector Zenil has basically proposed is to calculate the bits of Omega in the limit from below.

Robert J. Marks:

Oh my goodness. What an idea.

Gregory Chaitin:

This gives you a cryptocurrency where what you calculate is very, very useful. It's useful in the way that Marvin Minsky pointed out that it tells you the best theories for things. The most concise programs for things. That can be used for making predictions.

Robert J. Marks:

Well, one of the interesting things about this talking to George Gilder, who was kind of an economics guru and a forecast for the future says one of the problems with Bitcoin is the amount of value that you can get out of it is fixed. There's only a certain amount of gold that can be mined from Bitcoin software. If we use the Omega number as a basis for a cryptocurrency, there would be no limit, would there? It would just get harder and harder and harder to mine the gold.

Gregory Chaitin:

Right. The further you go, and the results of those computations are actually useful. Whereas in Bitcoin, you're spending a very, very large amount of computing power. Mining Bitcoins, and it's only useful for Bitcoins.

Robert J. Marks:

Yes.

Gregory Chaitin:

It's not useful for anything else. Hector thinks that's a terrible waste. At any rate, I think it's a... I'm not terribly interested in practical applications, not when I'm thinking about mathematics and Omega and everything, but the fact that it proposed as a new cryptocurrency, I think is fantastic. It's just a lot of fun. Who would have guessed?

Robert J. Marks:

It is really interesting.

Gregory Chaitin:

But you have to wait 60 years though. When I was publishing these papers, if I had to do practical work, I wouldn't have been able to do that research. 60 years later, this is practical research based on the Omega number.

Robert J. Marks:

That is just a fascinating idea. One of the, I think the poster problem for the Turing halting problem is Goldbach's Conjecture, which says that every even number can be expressed as the sum of two primes. If one had a halting problem or a halting Oracle, if you will, you could solve Goldbach's conjecture very easily by looking for a single counter example or showing that no counter example exists forever. Do you ever think that something like Chaitin's number can be calculated to a precision, which would allow for the proof or disproof of something like Goldbach's conjecture, which is a relatively short program?

Gregory Chaitin:

Yeah. It's relatively short as computer programs go, but there are a lot of programs up to that size. It grows exponentially. The calculation gets quite horrendous. The algorithm that extracts, given the *n* bits of Omega, that tells you for each of the programs up to *n* bits in size, which one halts and which one doesn't. If you do the obvious algorithm, its runtime is worse than super exponential. It grows as the busy beaver function of *n*.

Robert J. Marks:

Oh gosh.

Gregory Chaitin:

It looks tough. By the way, there's another interesting example of a famous mathematical problem called the twin prime conjecture that there are infinitely many prime numbers that are two consecutive odd numbers.

Robert J. Marks:

Yes.

Gregory Chaitin:

It certainly looks to be the case. In fact, there are very good estimates that seem empirically validated formulas that tell you the distribution of twin primes, how many there are. Can't be proven, but there's a formula which gets more and more accurate, the more twin primes you calculate. So it looks like they're infinitely many, and even we know the distribution, at least in the sense that an empirical scientist knows anything. But the question of whether there are infinitely many twin primes is not equivalent to a halting problem.

Robert J. Marks:

Yes, it can't be solved with a Turing Oracle code.

Gregory Chaitin:

Yeah, there is no finite counterexample. You can't have a program searching for a counterexample. Then if the program doesn't halt, you know there is no counter example. Omega doesn't solve all problems. Now there are extended versions of Omega. There's a hierarchy of Omegas that look at more and more abstract mathematical questions. Knowing the bits of Omega, wouldn't allow you to solve the twin prime conjecture positively or negatively, but there's a thing called the jump of the Omega number, that sort of would. You have Omega, Omega prime, Omega double prime.

Robert J. Marks:

Omega prime assumes that you have a halting Oracle?

Gregory Chaitin:

Exactly. That's correct.

Robert J. Marks:

You have this Omega prime that includes a halting Oracle, but if you have a halting Oracle, then you have to have a Meta halting Oracle that looks at regular computer programs with the regular halting Oracle. The Turing halting problem becomes more and more problematic. But in each case you have a more and more sophisticated things that you can do with Omega, which is just astonishing.

Gregory Chaitin:

But you would need an Oracle to do those things.

Robert J. Marks:

Yes. Those are probably not computable. I'm wondering if there's any way to get a handle on what Omega prime is.

Gregory Chaitin:

Well, a colleague of mine in Buenos Aires, Veronica Becher, and I mostly at her initiative worked on Omega prime and even on Omega double prime. Another way to define Omega prime is it's not the probability the program will halt, it's the probability that a program will produce a finite amount of output instead of producing an infinite amount of output. These are programs that never halt, or that may halt, or may not halt. A program that halts obviously only produces a finite amount of output. The property that the program generated at random produces an infinite amount of output, sorry, a finite amount of output is Omega prime and Omega double prime is the...

Gregory Chaitin:

You talk about programs calculating only one, two, three, four, five. It's the probability that a program produces all but a finite number of the positive integers. These are programs that calculate only zero, one, two, three, four, five. The question is the numbers that it doesn't calculate, will that be a finite number of numbers or will it be an infinite number of numbers? That's Omega double prime. This was work mostly done by Veronica Becher and students of hers, but I participated a little bit in that work. By now it's probably been generalized and extended in all directions, like happens in pure mathematics all the time.

Robert J. Marks:

Well, that's the thing. Omega itself is mind-blowing. But the fact that there's a regress into Omega prime and Omega double prime is hyper mind-blowing if you will. And it goes on and on, doesn't it?

Gregory Chaitin:

Yeah. Well this happens also. This hierarchy of things that are more and more unknowable, there's the normal computer. There's a computer that has an Oracle for the halting problem. Then there's a computer that has an Oracle for the computer that has an Oracle for the halting problem. It goes up and up. This is related to the... It's a sort of a hierarchy like Cantor's theory of infinities. There's infinity of the integers, infinity of the reals. For any infinity, there's a bigger infinity, which is the infinity of all subsets of the previous step. You have a hierarchy and dare I say that this is the notion maybe that you can only approach God in the limit. I think Cantor was interested in this question.

Robert J. Marks:

In fact, I believe that Cantor attempted to make a audience with Pope Leo XIII. I believe it was Pope Leo XIII about his theory of trans finite numbers. He never got to the Pope, but he got to one of his subordinates because he thought about the theological implications of his theory of infinite.

Gregory Chaitin:

Absolutely. I regard Cantor's theory of infinities as a mathematical theology. It's beautiful stuff. It's also paradoxical. Apparently Cantor thought that God was speaking to him. He felt he was inspired. In a way, anyone who is creative sounds like God is speaking to us. In mathematics that I would say is discovered not invented, you feel you're touching a reality beyond normal reality. This is in that BBC Four a 90-minute documentary, “Dangerous Knowledge”, talks about this. It was funded by the Templeton Foundation. It's a little -

Robert J. Marks:

Okay, I'm not familiar with this. What is the name of it?

Gregory Chaitin:

“Dangerous Knowledge.” If you go to my website, it's on Cantor, Boltzmann, Gödel and Turing. For a while, it wasn't available on the web or anywhere else, but it is now on the web. I hope the fact that I put this on my website doesn't mean that they're going to be forced to take it down. It's in five parts and I give the links on my website with the original poster that the BBC Four used. I think it's a wonderful movie. The gentleman who made it, David Malone actually went out to the places where all this work was done, to all these places.

Gregory Chaitin:

Also in some cases to the cemeteries where these amazing mathematicians, or philosophers or theologians, whatever they are - physicist in case of Boltzmann - are buried. Maybe it may be, it's not obvious that when he talks about one of these thinkers, he's actually showing you where he worked and maybe where he's buried and things like that. It's like a mathematical travel documentary. It's quite a fun film.

Robert J. Marks:

I've always found that interesting. We know what these people do, but not who they were. I think visiting the grave sites kind of tells us who they were.

Gregory Chaitin:

I visited Gödel and Von Neumann grave sites at Princeton.

Robert J. Marks:

You did. Oh they're buried in Princeton, I didn't know that.

Gregory Chaitin:

Buried in the cemetery. That's a short walk from Princeton University.

Robert J. Marks:

I see. Is Einstein buried there also, do you know?

Gregory Chaitin:

No. Einstein insisted that he be cremated and the ashes be scattered at an undisclosed location.

Robert J. Marks:

Really? Okay. I didn't know that.

Gregory Chaitin:

He didn't want to leave a shrine. I also visited Gödel’s home, which is near the cemetery. It's in a poor part of town. Von Neumann had a home that I never visited unfortunately. I only found out where it was, it's in the wealthy part of Princeton. They're together in the same cemetery, rather close to each other. Anyway, I visited Gödel’s home and the people who are renting it from the current owner, let me in. They showed me things that were left there from when Gödel was the inhabitant. For example, he had soundproofing put in because noise distracted him enormously. His wife had a shrine to the Virgin Mary in the backyard, which was still there. In fact, their home was in the neighborhood, which when they bought the home or Italian laborers, people who did construction work, I think.

Gregory Chaitin:

It was in a relatively modest part of town. Gödel didn't marry a professor's daughter. He married a woman he fell in love with, from comparatively humble origins. I think she felt more comfortable. Also, it was a Catholic neighborhood, whereas Princeton of course is some form of high Protestantism I believe, was at that time.

Robert J. Marks:

Yeah. Well certainly Princeton was founded on the Protestant ethic and boasted of a number of evangelical presidents for a while.

Gregory Chaitin:

That I didn't know.

Robert J. Marks:

Yeah. Well, that's true of a lot of the universities, they were founded based on Christianity, but they have strayed from that task as time goes on.

Gregory Chaitin:

Two notable examples are Oxford and Cambridge. Oxford was for teaching priests, Catholic priests and there was a schism. So a group went off and founded Cambridge.

Robert J. Marks:

Okay.

Gregory Chaitin:

This was a long time ago, probably something like 500 years ago.

Robert J. Marks:

That's fascinating. Well, Professor Chaitin, this has been, this has been a delight talking to you. We've been talking to Professor Gregory Chaitin today about a number named after him. We also went into some other topics, but Chaitin's number referred to as Omega. That's what he calls it and Cover and Thomas in their classic textbook "Elements of Information Theory" referred to it as Chaitin's magical, mystical number Omega. We did not have the time to go into the details of Chaitin's number, but we will give some links to some of Professor Chaitin's lectures, where he explains this incredible mind-blowing number. Until next time on Mind Matters News, be of good cheer.

Announcer:

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